Excalibur Primary School
Calculation Policy

Adopted: November 2022

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Sense of Number Maths Consultants

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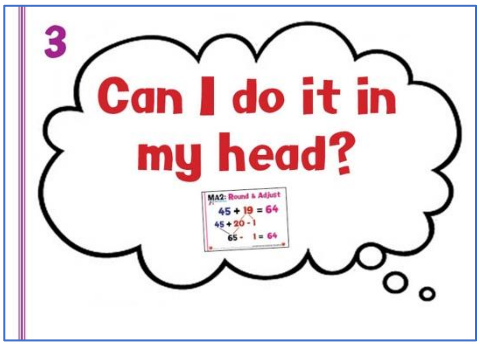
General Principles of Calculation

When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. Whatever method is chosen (in any year group), it must still be underpinned by a secure and appropriate knowledge of number facts.

By the end of Year 5, children should:

- have a secure knowledge of number facts and a good understanding of the four operations in order to:
    - carry out calculations mentally when using one-digit and two-digit numbers
    - use particular strategies with larger numbers when appropriate
- use notes and jottings to record steps and part answers when using longer mental methods
- **have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;**



Children should always **look at the actual numbers (not the size of the numbers)** before attempting any calculation to determine whether or not they need to use a written method.

Therefore, the key question children should always ask themselves before attempting a calculation is

‘***Can I do this in my head?’***

This should be the first principle for any calculation, whether addition, subtraction, multiplication & division. If a child is asking themselves this question it means that they are looking at the actual numbers and using their sense of number to determine whether there is an easier method that they could use rather than column / standard procedures.

In the previous curriculum, the next question that children should ask themselves (if they could answer the calculation mentally) would be whether or not they needed to jot something down to support their thinking and help them calculate – ‘***Do I need a jotting?’.*** If the answer to this question was ‘***yes’*** then the children would access the wide range of mental strategies & jottings outlined later in this document.

If, however, the numbers were too difficult or unwieldy for a mental method or jotting to be appropriate then they would ask the question

‘Do I need a written method?’. Again, if the written method was the most efficient and appropriate way to find the answer, they would access one of the written methods (either informal or standard) found later in this document.

(NB. Section 5 explains this principle in detail, with examples for each of the four operations) ```

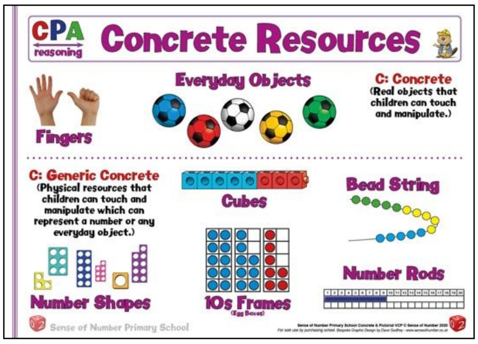
The 2014 National Curriculum, though, recommends a **Mastery** approach to teaching calculation, meaning that there are now 2 additional questions which children should be asking before moving to jottings or written methods – '***Can I make it?'*** and '***Can I draw a picture of it?'.***

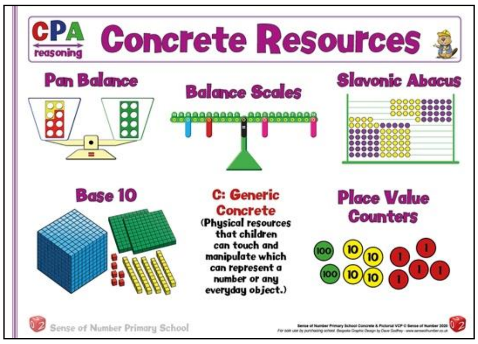
These are explained in the next section. (see below)

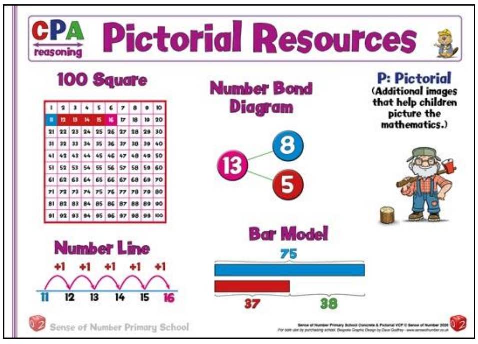
Concrete and Pictorial Resources / 'Mastery' of Mathematics

The ability of a child to work calculations out mentally, and also to successfully master a written procedure should initially come from a secure understanding of each calculation.

This is achieved through the use of concrete and pictorial resources throughout EYFS and both Key Stages.







The 2014 National Curriculum advocates a '**mastery'** approach to mathematics suggesting all children should master conceptual understanding of any calculation they are attempting to solve.

Mastery is the approach adopted in the Far East when teaching mathematics, (particularly in Singapore and Shanghai), and aims to ensure that children are given a substantial depth of understanding in place value for each year group.

This understanding, which is built up through regular use of concrete apparatus becomes the foundation of the **CPA (Concrete – Pictorial – Abstract)** approach to teaching.

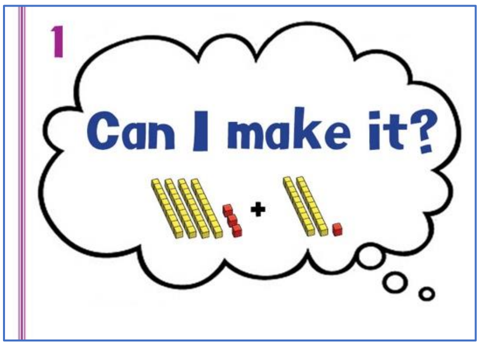
Children who have developed secure visual place value through the use of concrete apparatus, manipulatives and images (such as **Base 10 / Tens Frames / Number Rods / Numicon / Place Value Counters** etc) are then able to apply it when learning methods of calculation.

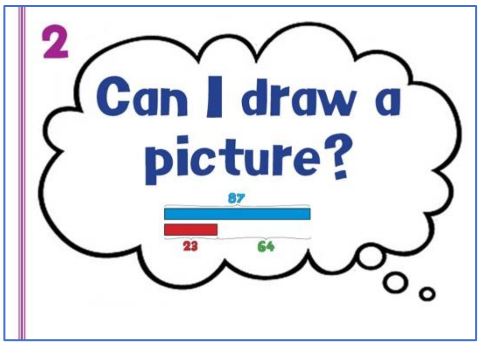
This ensures that they fully understand a calculation rather than just learning a method by rote. Consequently, the written method 'makes sense' and can be retained much more easily as it is based on conceptual understanding.

Therefore, a child who has 'mastered' a calculation can now ask themselves 2 extra questions that enable them to explain their understanding ('***Can I make it?'***)

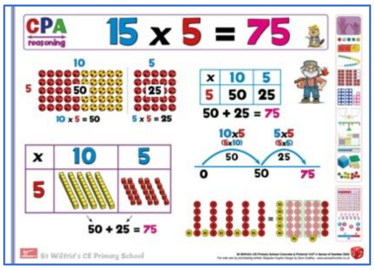
or give them a visual alternative to a written method

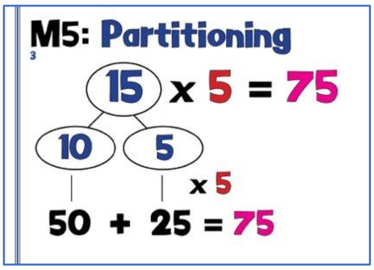
(***'Can I draw a picture?'***).

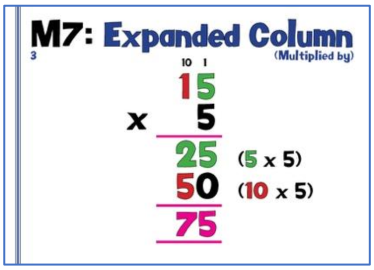


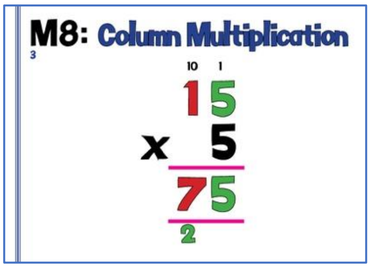


The example below shows the development of 15 x 5 from **Concrete** understanding (Place Value Counters and Base 10) to **mental jottings** (Number Line) to **informal methods** that support mental arithmetic (Grid Method / Partitioning) to an **expanded method** and finally to the **column procedure.**



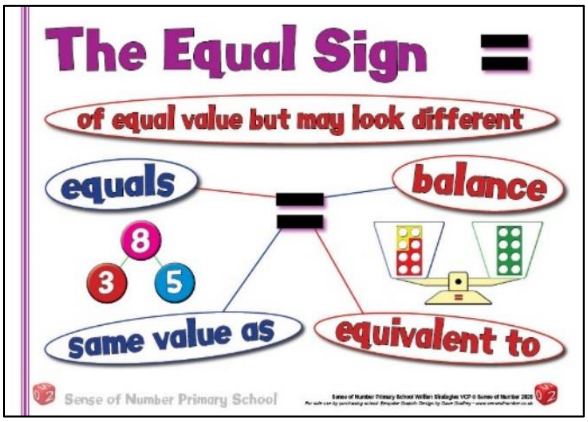






This calculation policy has been upgraded in line with the 2014 National Curriculum to reflect Mastery, and (as seen above) each calculation method now begins with Concrete materials before progressing to the mental and written methods advocated by the previous curriculum.

The Importance of Vocabulary in Calculation



It is vitally important that children are exposed to the relevant calculation vocabulary throughout their progression through the four operations.

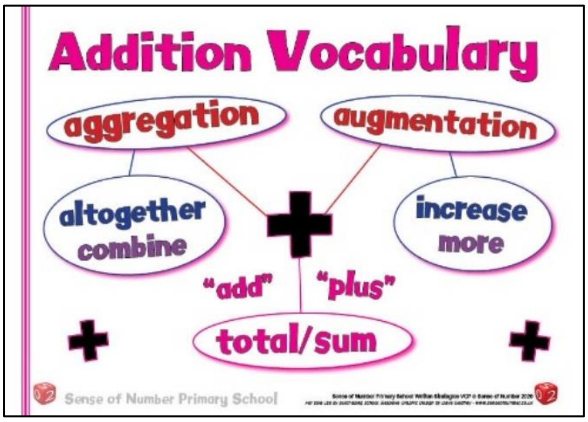
Initially, they need to understand the meaning of '**equals'**, which, in general terms means that the amount on either side of the equals symbol has the **same value, but may look different.** E.g. **20 + 4 = 8 x 3** or **½ of 96 = 50 – 2**

Equals can also be described and visualised as a '**balance'** or used to determine when amounts are **equivalent.**

Key Vocabulary:

(to be used from **Y1** for **addition & subtraction** and **Y2** for **multiplication & division**)

Each of the four number operations has key associated vocabulary, as well as specific structures, models and images. Each of these will be discussed, pictured and outlined later in this document in the appropriate sections.



Addition: The detailed discussion of vocabulary and associated images and structures (**aggregation & augmentation**) for addition will be discussed in the section on **Addition Progression** (Pages ???)

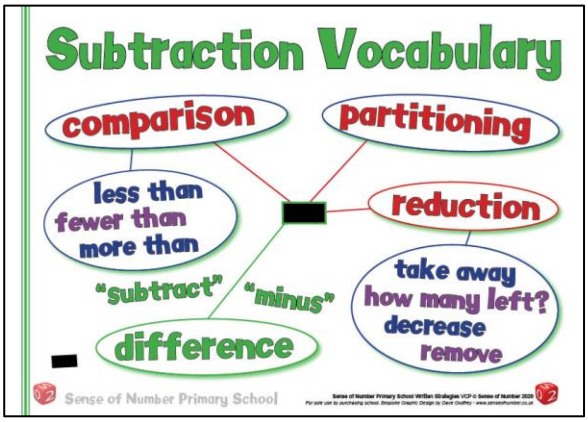
The main **language** of **addition**, which can be used for any calculation, no matter what structure is being used, is: -

Total & Sum **Add / Plus**

E.g. 'The **sum** of 12 and 4 is 16', '12 **add** 4 equals 16'

'12 and 4 have a **total** of 16' '12 **plus** 4 equals 16'

The other words on the poster (**altogether, combine, increase, more)** are more specific to different structures, and will be discussed later.



Subtraction: The detailed explanation of subtraction vocabulary and images, linked to the 3 key structures of **comparison, partitioning** and **reduction**, will be discussed in the section on **Subtraction Progression** (Pages ???)

The main **subtraction vocabulary**, which can be used for any calculation, no matter which key structure, is: -

Difference between

Subtract / Minus **(not 'take away')**

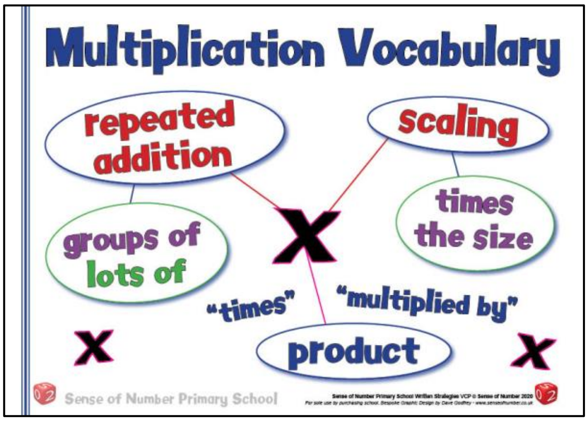
E.g. 'The **difference between** 12 and 4 is 8',

'12 **subtract** 4 equals 8'

'12 **minus** 4 equals 8'

The other words on the poster (**less than, fewer than, more than, take away, how many left, decrease, remove)** are more specific to different structures, and will be discussed later.

Multiplication: Multiplication vocabulary and imagery, and its associated structures of **repeated addition** and **scaling**, will be explored in greater detail later in the document (Page ???)



The key **vocabulary** for **multiplication**, which can be used for any calculation, no matter which key structure, is:

Product **Multiplied by / Times**

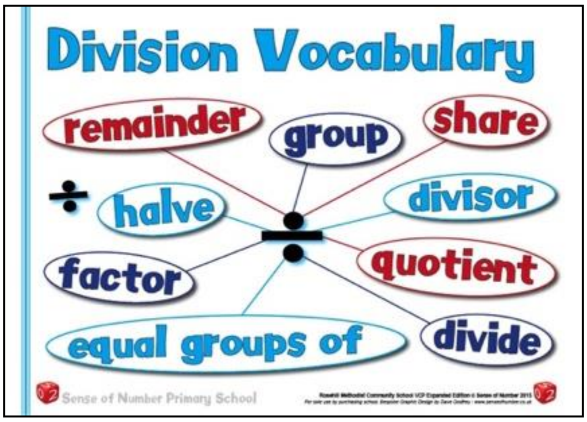
E.g. 'The **product** of 12 and 4 is 48',

'12 **multiplied** by 4 equals 48'

12 **times** 4 equals 48

The words on the poster (**groups of, lots of, times the size),** which are more specific to different structures, will be discussed later.

Division: There will be a more detailed explanation of division, including the two key structures and the main models and images that support division, later in the document (Page ???)



Divisor & Quotient **Divide**

E.g. 'The quotient of 12 and 4 is 3',

'12 divided by 4 equals 3'

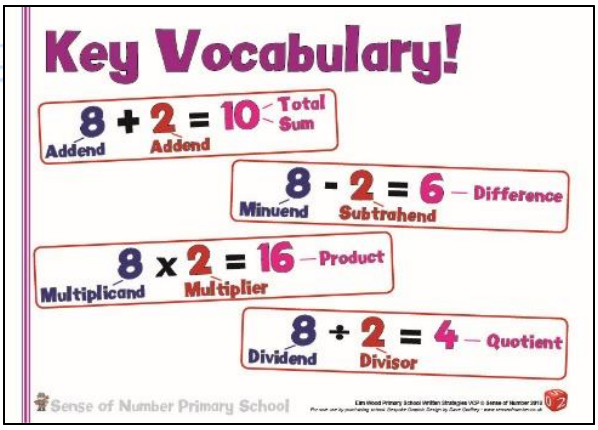
'When we divide 12 by 4, the divisor of 4 goes into 12 three times'

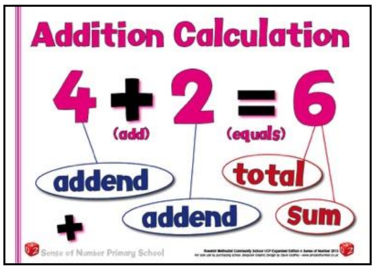
Additional Vocabulary:

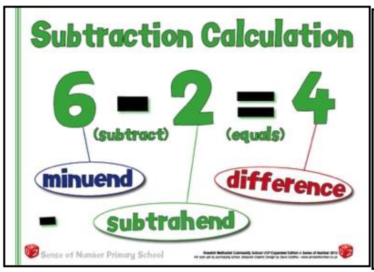
The VCP vocabulary posters contain both the key and additional vocabulary children should be exposed to.

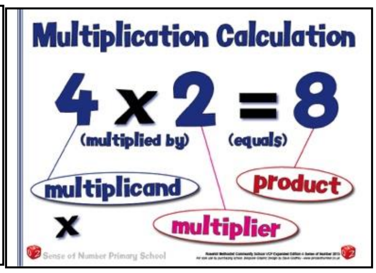
Conceptual Understanding

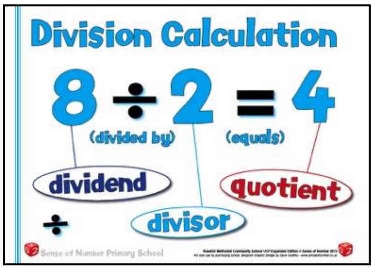
Using key vocabulary highlights some important conceptual understanding in calculation. For example, the answer in a subtraction calculation is called the difference. Therefore, whether we are counting back (taking away), or counting on, to work out a subtraction calculation, either way we are always finding the difference between two numbers.











Mental Methods of Calculation

Oral and mental work in mathematics is essential, particularly so in calculation.

Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts.

Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

On-going oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learnt to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a **'sense' of number** is the product of structured practice and repetition. It requires an understanding of **number patterns and relationships** developed through **directed enquiry,** use of **models and images** and the application of **acquired number knowledge and skills**. Secure mental calculation requires the ability to:

- ***recall key number facts instantly*** – for example, all number bonds to 20, and doubles of all numbers up to double 20 **(Year 2)** and multiplication facts up to 12 × 12 **(Year 4);**
- **use taught strategies to work out the calculation** – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number to a one-digit or two-digit number **(Year 1),** add two-digit numbers in different ways **(Year 2),** add and subtract numbers mentally with increasingly large numbers **(Year 5);**
- understand how the rules and laws of arithmetic are used and applied – for example to use commutativity in multiplication **(Year 2),** estimate the answer to a calculation and use inverse operations to check answers **(Years 3 & 4),** use their knowledge of the order of operations to carry out calculations involving the four operations **(Year 6).**

The first 'answer' that a child may give to a mental calculation question would be based on instant recall.

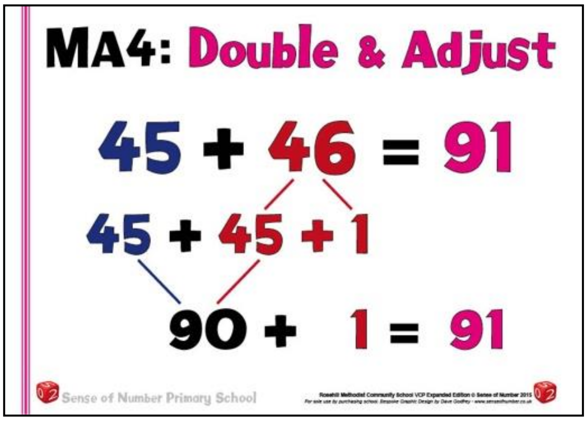
E.g. "What is 12 + 4?", "What is 12 x 4?", "What is 12 – 4?" or "What is 12÷ 4?" giving the immediate answers "16", "48", "8" or "3"

Other children would still work these calculations out mentally by counting on from 12 to 16, counting in 4s to 48, counting back in ones to 8 or counting up in 4s to 12.

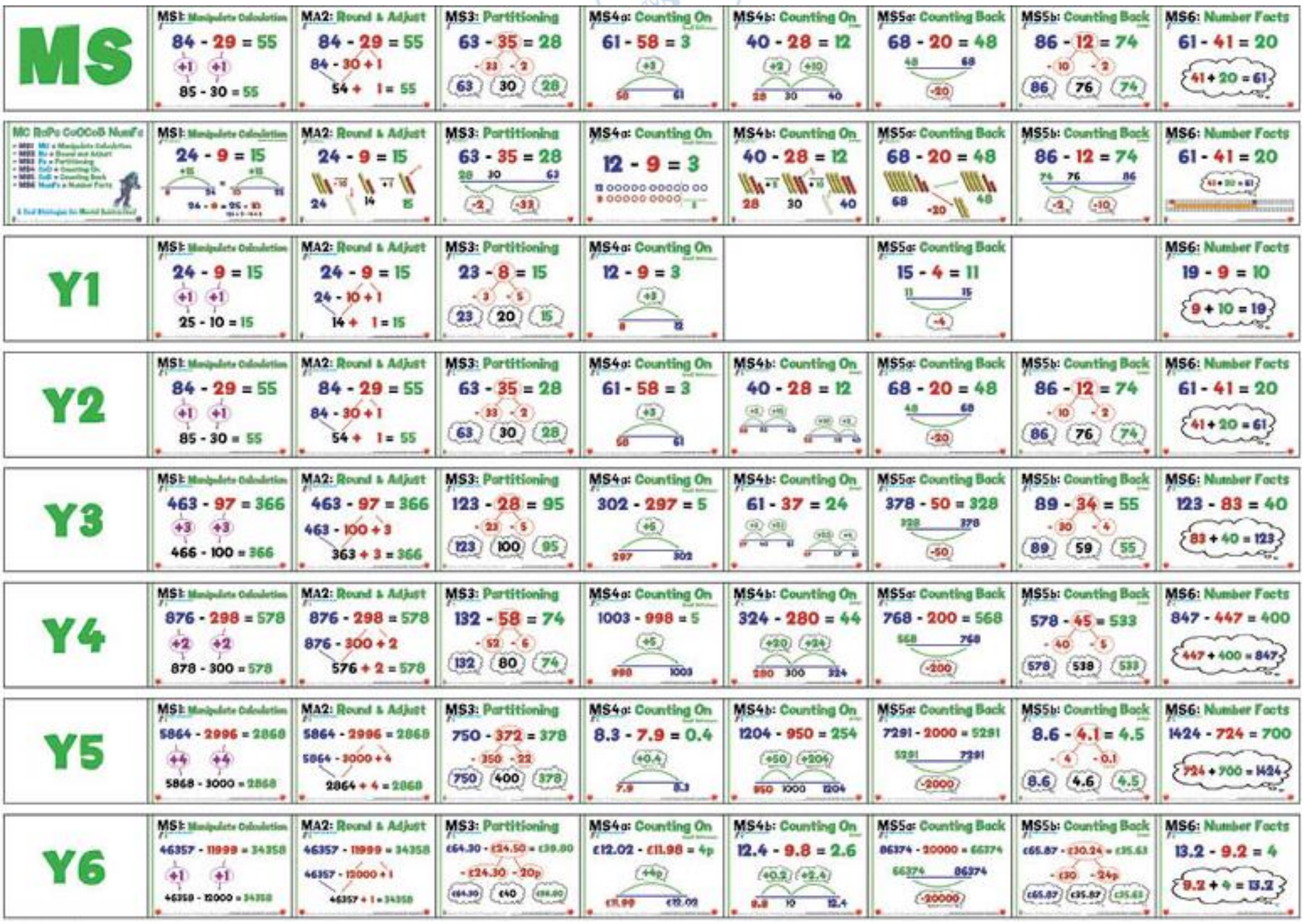
From instant recall, children then develop a bank of mental calculation strategies for all four operations, in particular addition and multiplication.

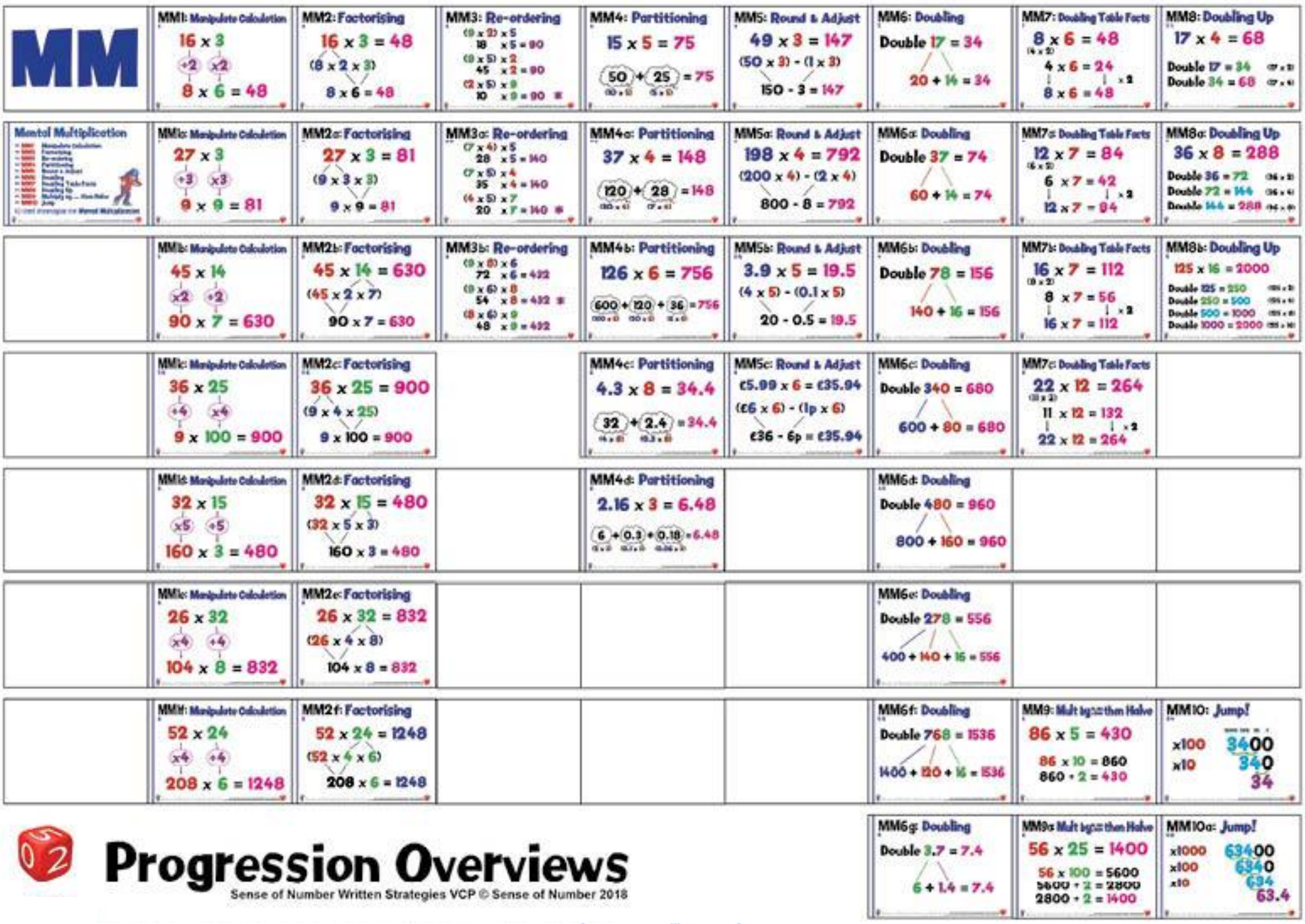
These would be practised regularly until they become refined, where children will then start to see and use them as soon as they are faced with a calculation that can be done mentally.

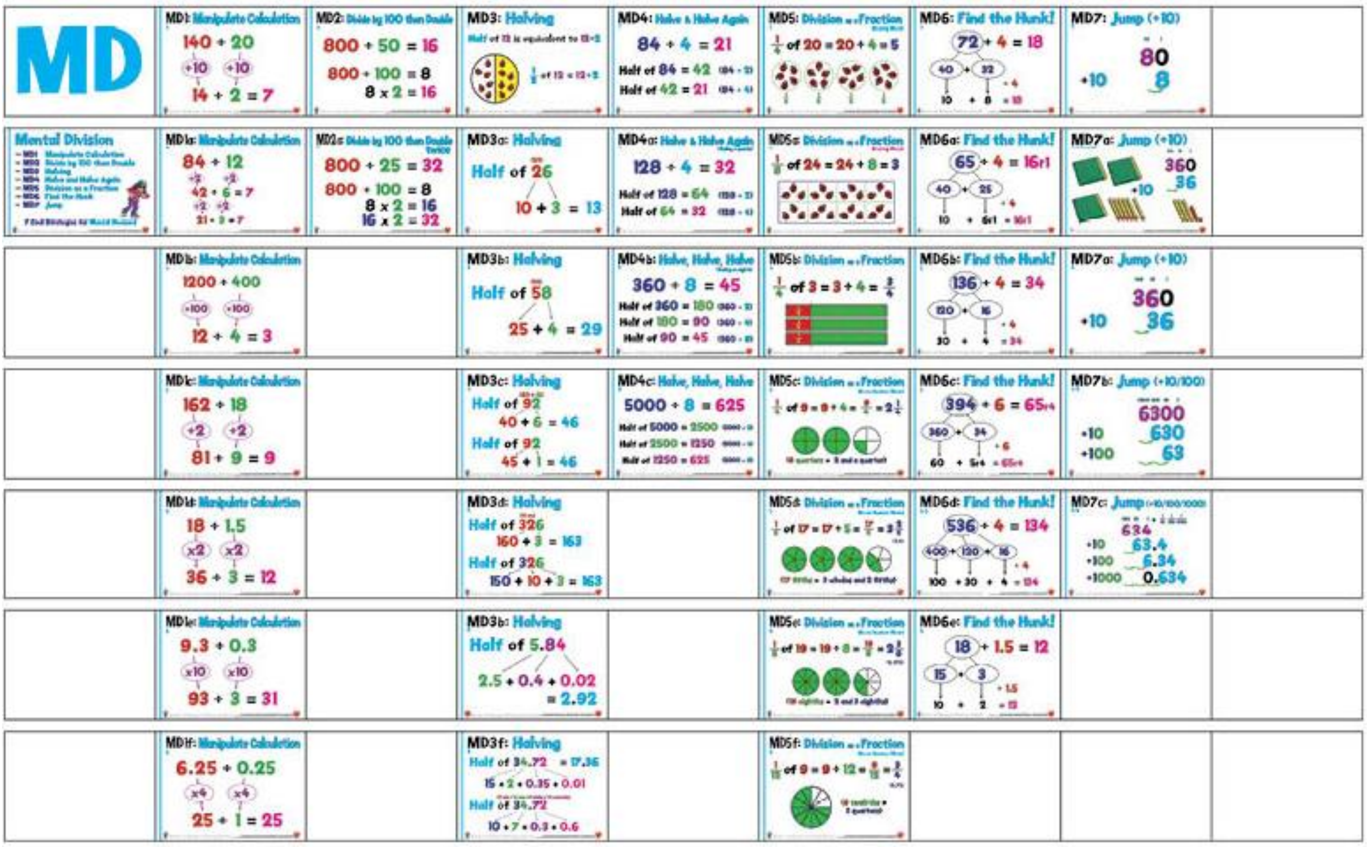
The following pages contain thumbnails of the posters found in the Mental Calculation Policy.




MA
MA1: Manipulate Calculation
35 + 19 = 54
34 + 20 = 54
MA2: Round & Adjust
35 + 19 = 54
35 + 20 - 1
55 - 1 = 54
MA3: Partitioning
35 + 82 = 117
110 + 7 = 117
MA4a: Counting On
+5 +3
35 + 8 = 43
MA4b: Counting On
35 + 20 = 55
+20
MA5: Double & Adjust
35 + 36 = 71
70 + 1 = 71
MA6: Number Bonds
35 + 95 = 130
30 + 100 = 130
MC RaPa CoDa Numbo
16 + 9 = 25
15 + 10
45 + 9 = 54
43 + 21 = 64
8 + 6 = 14
45 + 20 = 65
7 + 8 = 15
7 + 8 = 7 + 7 + 1 = 14 + 1 = 15
3 + 4 + 7 = 14
Y1
16 + 9 = 25
15 + 10 = 25
45 + 9 = 54
45 + 10 - 1 =
55 - 1 = 54
8 + 6 = 14
8 + 2 + 4 = 14
+2 +4
8 + 6 = 14
57 + 10 = 67
+10
5 + 6 = 11
10 + 1 = 11
3 + 4 + 7 = 14
10 4
Y2
45 + 19 = 64
44 + 20 = 64
45 + 19 = 64
45 + 20 - 1
65 - 1 = 64
43 + 21 = 64
60 + 4 = 64
+2 +5
78 + 7 = 85
58 + 40 = 98
+40
7 + 8 = 15
14 + 1 = 15
13 + 4 + 7 + 16 = 40
20 20
Y3
45 + 97 = 142
42 + 100 = 142
45 + 97 = 142
45 + 100 - 3
145 - 3 = 142
MA1: Partitioning
57 + 25 = 82
70 + 12 = 82
+20 +30
85 + 50 = 135
534 + 300 = 834
+300
16 + 17 = 33
32 + 1 = 33
42 + 16 + 28 + 54 = 140
70 70
Y4
345 + 298 = 643
343 + 300 = 643
345 + 298 = 643
345 + 300 - 2
645 - 2 = 643
648 + 231 = 879
800 + 70 + 9 = 879
+20 +40
784 + 60 = 844
4837 + 3000 = 7837
+3000
37 + 38 = 75
74 + 1 = 75
342 + 16 + 28 + 114 = 50
370 130
Y5
4645 + 1996 = 6641
4641 + 2000 = 6641
4645 + 1996 = 6641
4645 + 2000 - 4
6645 - 4 = 6641
576 + 258 = 834
700 + 120 + 14 = 834
+200 +300
837 + 500 = 1337
7583 + 5000 = 12583
+5000
125 + 127 = 252
250 + 2 = 252
£4.56 + £3.27 + £1.44 = £9.27
£6.00 £3.27
Y6
45.2 + 49.9 = 95.1
45.1 + 50 = 95.1
45.2 + 49.9 = 95.1
45.2 + 50 - 0.1
95.2 - 0.1 = 95.1
4.73 + 2.21 = 6.94
6 + 0.9 + 0.04 = 6.94
+0.3 +0.5
6.7 + 0.8 = 7.5
5,763,947 + 4,000,000 = 9,763,947
+4,000,000
4.5 + 4.7 = 9.2
9 + 0.2 = 9.2
24.25 + 31.63 + 21.75 = 77.63
46 31.63


MS
MS1: Manipulate Calculation
84 - 29 = 55
85 - 30 = 55
MS2: Round & Adjust
84 - 29 = 55
84 - 30 + 1
54 + 1 = 55
MS3: Partitioning
63 - 35 = 28
MS4a: Counting On
61 - 58 = 3
MS4b: Counting On
40 - 28 = 12
MS5a: Counting Back
68 - 20 = 48
MS5b: Counting Back
86 - 12 = 74
MS6: Number Facts
61 - 41 = 20
41 + 20 = 61
24 - 9 = 15
12 - 9 = 3
Y1
24 - 9 = 15
25 - 10 = 15
24 - 10 + 1
14 + 1 = 15
23 - 8 = 15
12 - 9 = 3
15 - 4 = 11
19 - 9 = 10
9 + 10 = 19
Y2
84 - 29 = 55
85 - 30 = 55
84 - 30 + 1
54 + 1 = 55
63 - 35 = 28
61 - 58 = 3
40 - 28 = 12
68 - 20 = 48
86 - 12 = 74
61 - 41 = 20
41 + 20 = 61
Y3
463 - 97 = 366
466 - 100 = 366
463 - 100 + 3
363 + 3 = 366
123 - 28 = 95
302 - 297 = 5
61 - 37 = 24
378 - 50 = 328
89 - 34 = 55
123 - 83 = 40
83 + 40 = 123
Y4
876 - 298 = 578
878 - 300 = 578
876 - 300 + 2
576 + 2 = 578
132 - 58 = 74
1003 - 998 = 5
324 - 280 = 44
768 - 200 = 568
578 - 45 = 533
847 - 447 = 400
447 + 400 = 847
Y5
5864 - 2996 = 2868
5868 - 3000 = 2868
5864 - 3000 + 4
2864 + 4 = 2868
750 - 372 = 378
8.3 - 7.9 = 0.4
1204 - 950 = 254
7291 - 2000 = 5291
8.6 - 4.1 = 4.5
1424 - 724 = 700
724 + 700 = 1424
Y6
46357 - 11999 = 34358
46358 - 12000 = 34358
46357 - 12000 + 1
46357 + 1 = 34358
£64.30 - £24.50 = £39.80
£12.02 - £11.98 = 4p
12.4 - 9.8 = 2.6
86374 - 20000 = 66374
£65.87 - £30.24 = £35.63
13.2 - 9.2 = 4
9.2 + 4 = 13.2


MM
Mental Multiplication
MM1: Manipulate Calculation
MM2: Factorising
MM3: Re-ordering
MM4: Partitioning
MM5: Round & Adjust
MM6: Doubling
MM7: Doubling Table Facts
MM8: Doubling Up
MM1a: Manipulate Calculation
MM2a: Factorising
MM3a: Re-ordering
MM4a: Partitioning
MM5a: Round & Adjust
MM6a: Doubling
MM7a: Doubling Table Facts
MM8a: Doubling Up
MM1b: Manipulate Calculation
MM2b: Factorising
MM3b: Re-ordering
MM4b: Partitioning
MM5b: Round & Adjust
MM6b: Doubling
MM7b: Doubling Table Facts
MM8b: Doubling Up
MM1c: Manipulate Calculation
MM2c: Factorising
MM4c: Partitioning
MM5c: Round & Adjust
MM6c: Doubling
MM7c: Doubling Table Facts
MM1d: Manipulate Calculation
MM2d: Factorising
MM4d: Partitioning
MM6d: Doubling
MM1e: Manipulate Calculation
MM2e: Factorising
MM6e: Doubling
MM1f: Manipulate Calculation
MM2f: Factorising
MM6f: Doubling
MM9: Mult by 10 then Halve
MM10: Jump!
MM6g: Doubling
MM9a: Mult by 100 then Halve
MM10a: Jump!
Progression Overviews
Sense of Number Written Strategies VCP © Sense of Number 2018


MD
Mental Division
MD1: Manipulate Calculation
MD2: Divide by 100 then Double
MD3: Halving
MD4: Halve & Halve Again
MD5: Division as a Fraction
MD6: Find the Hunk!
MD7: Jump (÷10)
MD1a: Manipulate Calculation
MD2a: Divide by 100 then Double Twice
MD3a: Halving
MD4a: Halve & Halve Again
MD5a: Division as a Fraction
MD6a: Find the Hunk!
MD7a: Jump (÷10)
MD1b: Manipulate Calculation
MD3b: Halving
MD4b: Halve, Halve, Halve
MD5b: Division as a Fraction
MD6b: Find the Hunk!
MD7a: Jump (÷10)
MD1c: Manipulate Calculation
MD3c: Halving
MD4c: Halve, Halve, Halve
MD5c: Division as a Fraction
MD6c: Find the Hunk!
MD7b: Jump (÷10/100)
MD1d: Manipulate Calculation
MD3d: Halving
MD5d: Division as a Fraction
MD6d: Find the Hunk!
MD7c: Jump (÷10/100/1000)
MD1e: Manipulate Calculation
MD3e: Halving
MD5e: Division as a Fraction
MD6e: Find the Hunk!
MD1f: Manipulate Calculation
MD3f: Halving
MD5f: Division as a Fraction

CPA & Informal Written Methods / Mental Jottings

The ***2014 Curriculum for Mathematics*** sets out progression in the compact written methods of calculation for each of the four operations.

It also places some emphasis on the need to *'add and subtract numbers mentally'* (**Years 2 & 3**), mental arithmetic *'with increasingly large numbers'* (**Years 4 & 5)** and *'mental calculations with mixed operations and large numbers'* (**Year 6**).

There is very little guidance, however, on the 'jottings' and informal methods that support mental calculation, and which provide a clear link between answering a calculation entirely mentally (without anything written down) and completing a formal written method with larger numbers.

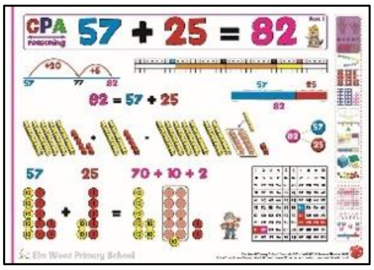
Although the 2014 Curriculum, as mentioned previously, also advocates the Mastery / CPA (**Concrete – Pictorial – Abstract**) approach to teaching calculation, it again gives almost no guidance as to what this approach would look like in the classroom.

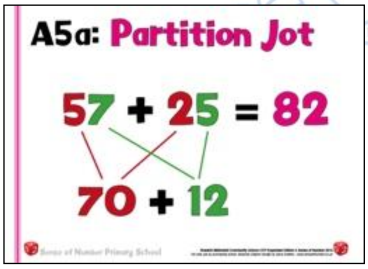
This policy, for all four operations, provides very clear guidance not only as to the development of formal written methods, but also: -

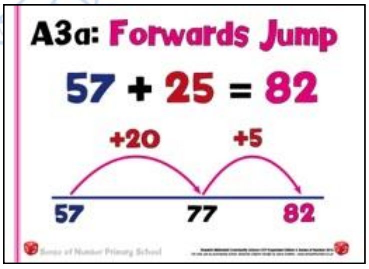
- some of the key concrete materials and apparatus that can be used in the classroom to support visualisation and depth of understanding for many of the calculation methods
- the jottings, expanded & informal methods of calculation that embed a sense of number and understanding before column methods are taught.
- A wide range of mental arithmetic strategies, not only for the more 'natural' addition and multiplication methods, but also subtraction and division.

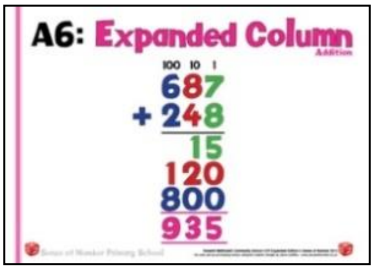
These valuable strategies include:

Addition:





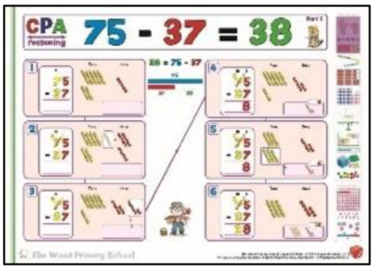


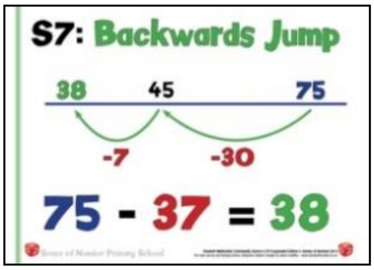


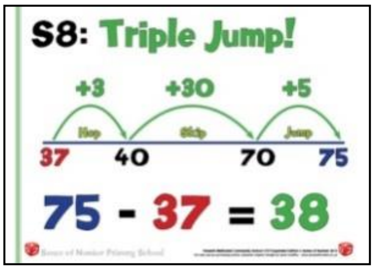
Concrete Materials | **Partitioning** | **Expanded Methods** | **Number Lines**

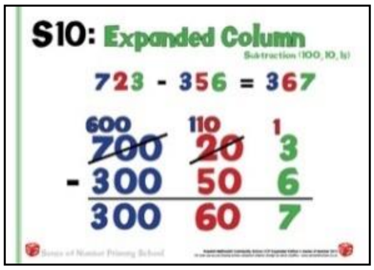
*(In addition to the 6 key mental strategies for addition - see '**Addition Progression'**)*

Subtraction:





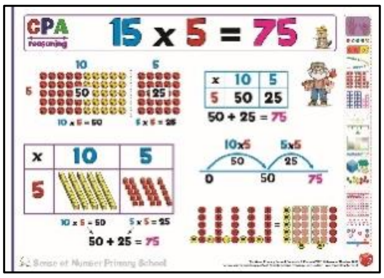


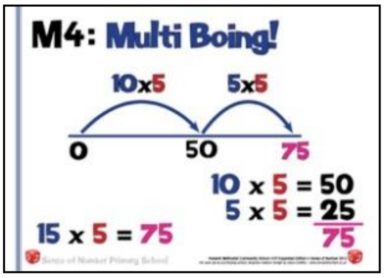


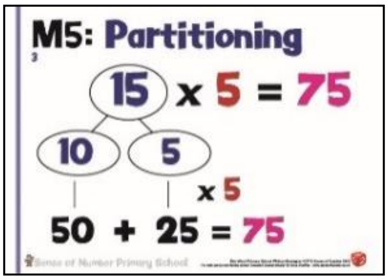
Concrete Materials | **Number Lines (especially for counting on)** | **Expanded Subtraction**

*(In addition to the 6 key mental strategies for subtraction - see '**Subtraction Progression')***

Multiplication:



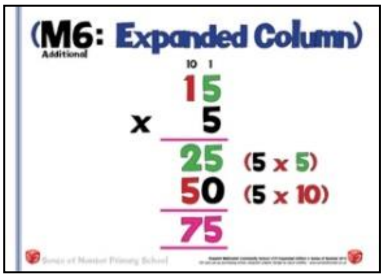


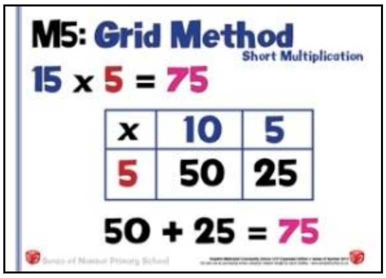


Concrete Materials

Number Lines

Partitioning



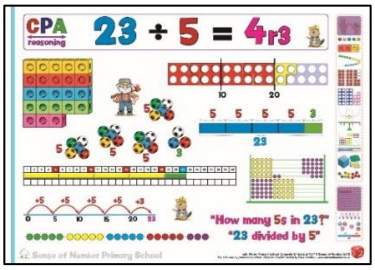


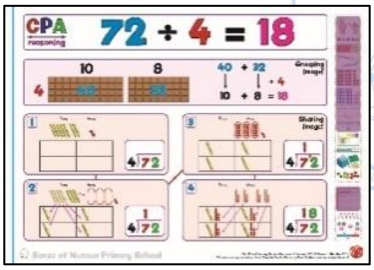
Expanded Method

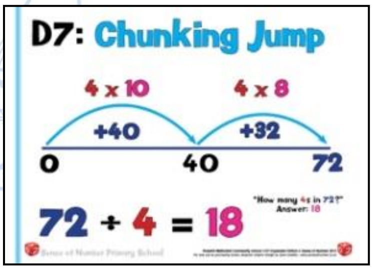
Grid Method

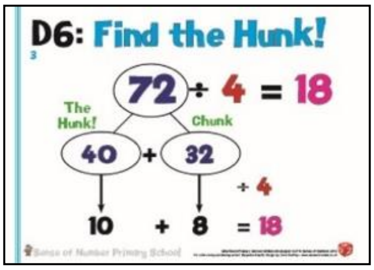
(In addition to the 10 key mental strategies for multiplication (see '***Multiplication Progression*)**

Division:









Concrete Materials

Number Lines

Chunking

(In addition to the 7 key mental strategies for division (see '***Division Progression'*)**

Formal (Column) Written Methods of Calculation

The aim is that by the end of **Year 5**, the great majority of children should be able to ***use an efficient written method for each operation with confidence and understanding with up to 4 digits***.

This guidance promotes the use of what are commonly known as ‘standard’ written methods – methods that are efficient and work for any calculation, including those that involve whole numbers or decimals. They are compact & consequently help children to keep track of their recorded steps.

Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that schools adopt greater consistency in their approach to calculation that all teachers understand and towards which they work.

There has been some confusion previously in the progression towards written methods and for too many children the staging posts along the way to the more compact method have instead become end points. While this may represent a significant achievement for some children, the great majority are entitled to learn how to use the most efficient methods.

The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

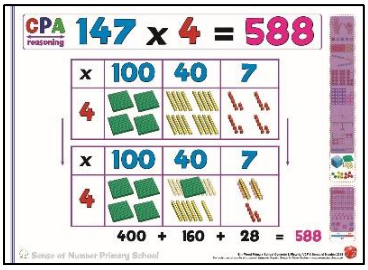
The incidence of children moving between schools and localities is very high in some parts of the country. Moving to a school where the written method of calculation is unfamiliar and does not relate to that used in the previous school can slow the progress a child makes in mathematics. There will be differences in practices and approaches, which can be beneficial to children. ***However, if the long-term aim is shared across all schools and if expectations are consistent then children’s progress will be enhanced rather than limited***.

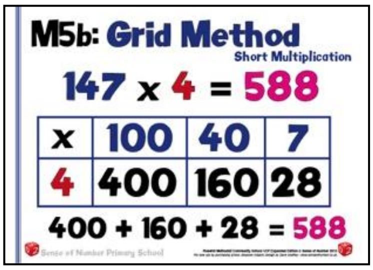
The entitlement to be taught how to use efficient written methods of calculation is set out clearly in the National Curriculum objectives. Children should be equipped to decide when it is best to use a mental or written method based on the knowledge that they are in control of this choice as they are able to carry out all methods with confidence.

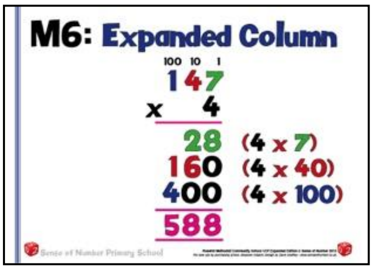
This policy does, however, clearly recognise that whilst children should be taught the efficient, formal written calculation strategies***, it is vital that they have exposure to models and images (CPA), and have a clear conceptual understanding of each operation and each strategy.***

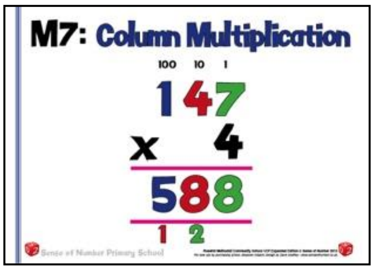
The visual slides that feature below (in the separate progression documents) for all four operations have been taken from the Sense of Number Visual Calculations Policy.

They show, wherever possible, the different strategies for calculation exemplified with identical values. This allows children to compare different strategies and to ask key questions, such as, ‘***what’s the same, what’s different?’***

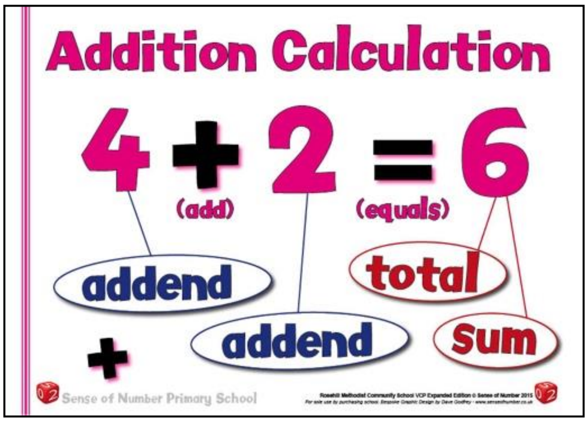








Addition Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

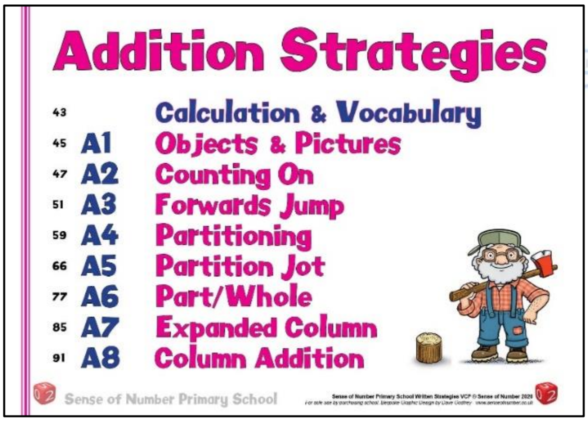
Children need to acquire **one efficient written method of calculation** for addition that they know they can rely on **when mental methods are not appropriate.**

To add successfully, children need to be able to:

- **recall all addition pairs to 9 + 9 and complements in 10;**
- **add mentally a series of one-digit numbers, such as 5 + 8 + 4;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**

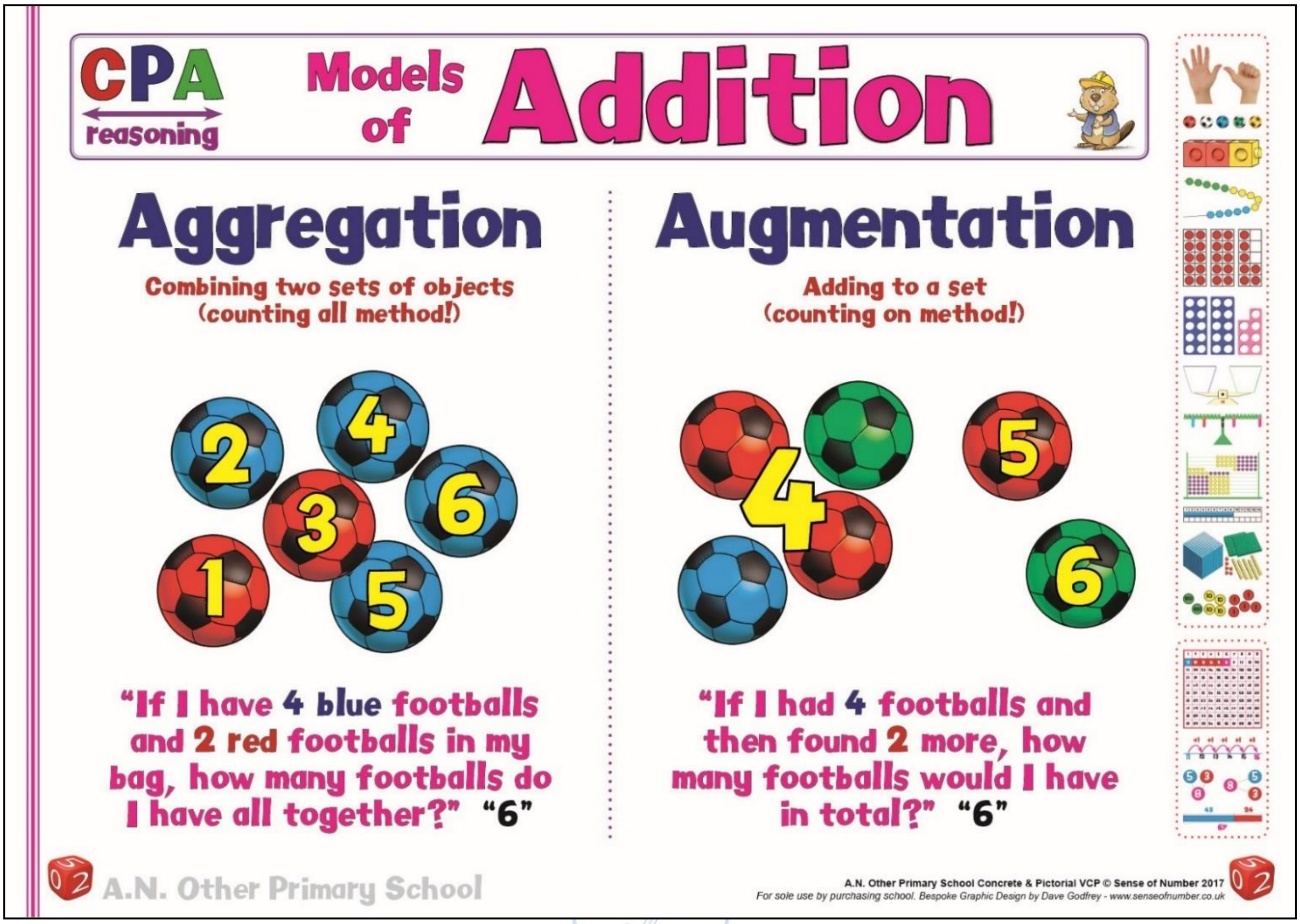
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Addition Strategies



There are 8 key strategies for jottings / written addition, which support the children's understanding, and which can be developed across the year groups. These can be seen on the left hand poster, and are explained in much more detail, with specific examples outlined, in the remainder of this section on written addition.

Models of Addition



As the images above show, it is important that children are introduced to the two main models for addition using practical resources.

In EYFS this would be real objects such as footballs, shells, cakes, cars, dinosaurs etc.

In KS1 this would progress from concrete materials such as counters or cubes to pictorial models and sketches.

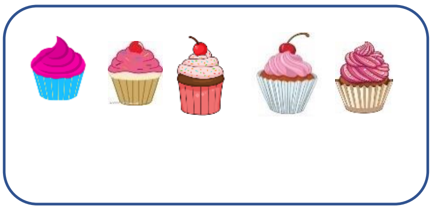
The two models of addition, **aggregation** and **augmentation** will develop into the two main mental / written methods of **partitioning** (aggregation) and **counting on** (augmentation).

Each of the addition models also have associated subtraction models. **Aggregation** is the **inverse** of subtraction by **partitioning**, while **augmentation** is the **inverse** of subtraction by **reduction.**

Aggregation

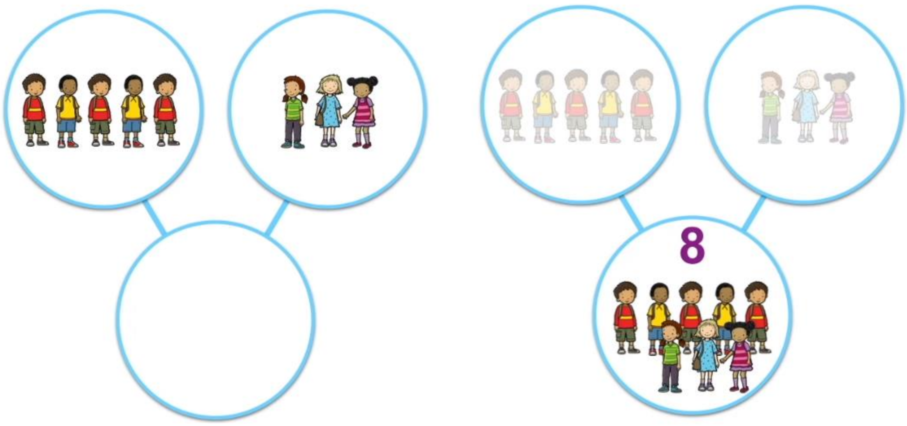
In simplistic terms, **aggregation** involves 2 separate groups that are combined to give an 'aggregate' total. E.g. There are 7 cakes in the first box and 5 cakes in the second box. How many cakes are there altogether?

7 + 5 = 12



Sometimes the two amounts can remain separate, as with the example above.

Sometimes they can be brought together, as with the example below



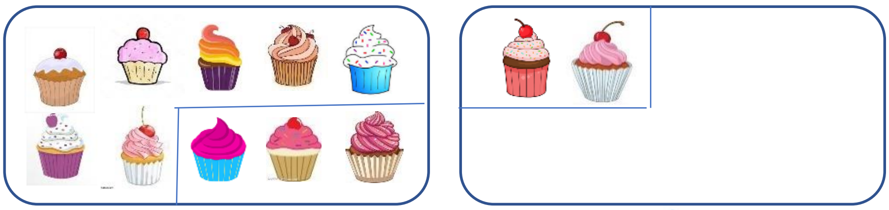
Aggregation

In the 'story' on the left, there are 5 boys and 3 girls in a group, making 8 children altogether.

The two parts are **'5'** and **'3',** the whole is **'8'**

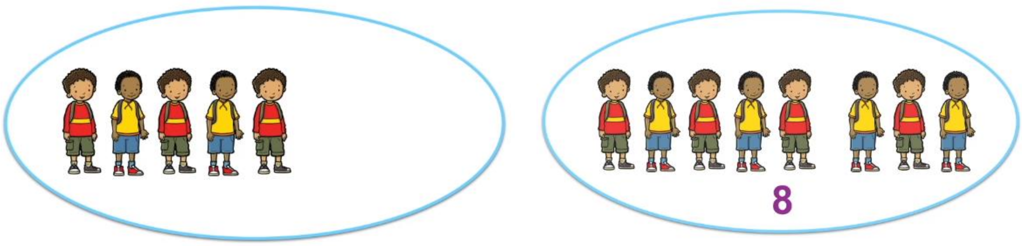
Augmentation

Augmentation, however, involves adding on to an existing group. E.g. There are 7 cakes in a box. My friend gives me 5 more cakes. How many cakes do I have now?



7 + 5 = 12

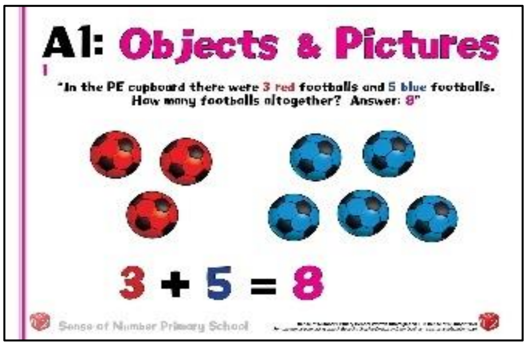
In the example above, an initial group of 7 cakes has been increased by 5.
In the example below, there were 5 children then 3 more joined them.

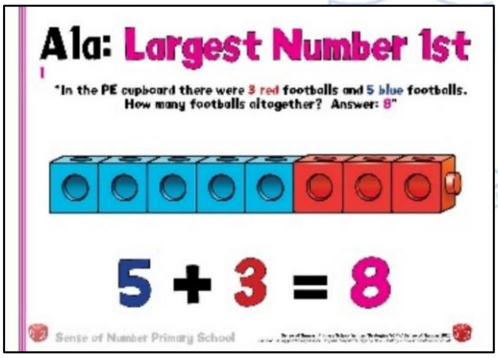


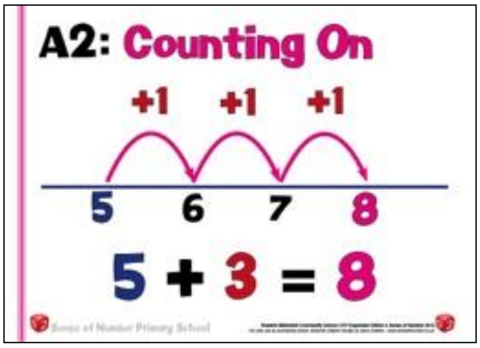
Augmentation

Both of these principles are explored in the actual written calculation policy below.

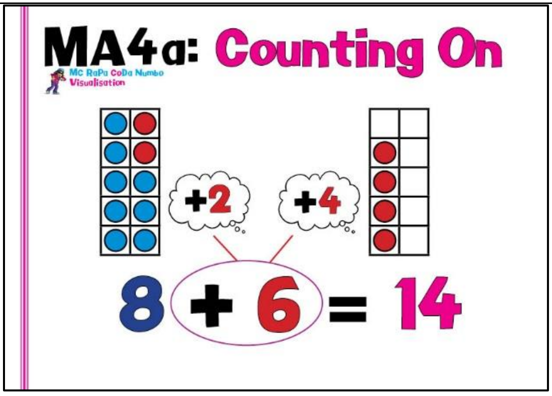
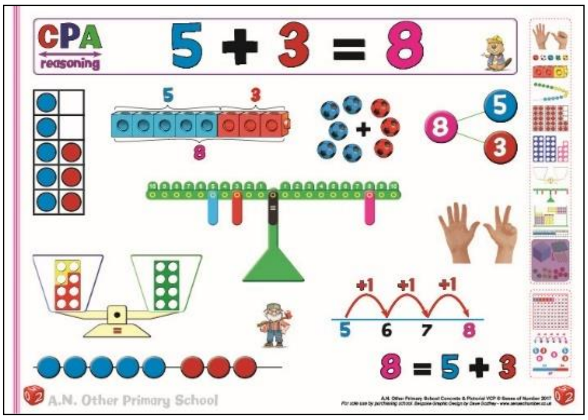
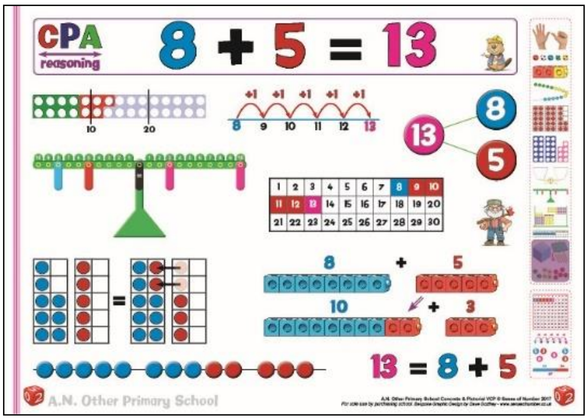
Written Methods of Addition – Finding The Total

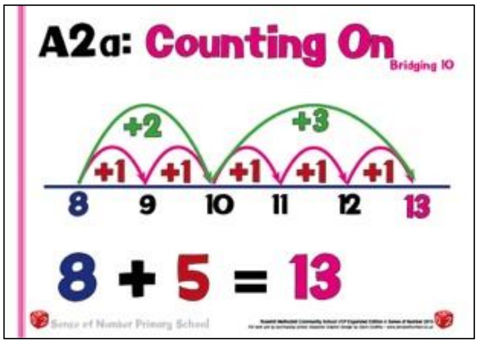
<table>
<tr><th>Stage 1</th><th>Aggregation
(Part-Whole Method –
How many altogether?)</th><th>Augmentation
(Counting On / Increasing the amount)</th></tr>
<tr><td>FS/Y1</td><td>Initially, children need to represent addition using a range of different resources and understand that a total can be found by counting out the first number, counting out the second number then counting how many there are altogether.</td><td></td></tr>
<tr><td></td><td>

This is aggregation (see above)</td><td>3 (held in head) then use fingers to count on 5
("3… 4,5,6,7,8)</td></tr>
<tr><td></td><td>

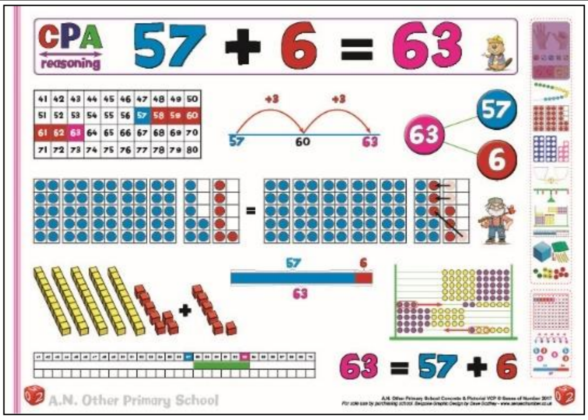
This will quickly develop into placing the largest number first, either as a pictorial / visual method or by using a number line.
The use of the number line, however, should be delayed until the children are completely secure in their understanding of 5 + 3. Otherwise it becomes a tool that limits their understanding of what they are actually representing
The word problem is also aggregation, although the image can be used for augmentation as well.</td><td>

5 (held in head)
then count on 3
(“5 … 6, 7, 8”)</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

<table>
<tr>
<td></td>
<td>
</td>
<td>The ‘Egg Box’ / ‘Ten Frame’ image is also an excellent visual tool to support augmentation</td>
</tr>
<tr>
<td></td>
<td>
</td>
<td>Before moving onto jottings or written methods (for any calculation), children need to be shown a wide range of images that support their understanding.
For simple calculations, the ‘equals’ sign can be viewed as a way to show the total (egg boxes, cubes, footballs, fingers) but also as a ‘balance’ (Balance scales, peg balance), where 5 and 3 have a value that is equal to / the same as, or that balances 8.</td>
</tr>
<tr>
<td>Y1/2</td>
<td>

When bridging through 10, a calculation can be seen as a straightforward ‘count on’ (see number line & 100 square images), a balance image as before (see balance & number bond diagram) but, more importantly as a strategy where the ‘5’ is partitioned into 2 and 3 (Numicon, multi-link and Tens Frames).
In effect, for these images / strategies, the calculation is being re-written as
8 + 5 = (8 + 2) + 3 = 10 + 3 = 13</td>
<td>

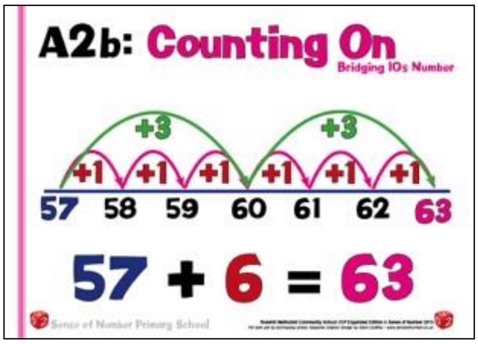
When developing the concrete picture into a jotting the number line can be used to display the ‘count on’
(8 + 1 + 1 + 1 + 1 + 1)
or the partition (8 + 2 + 3).
Once this image is secure, the same strategies can be done mentally: -
8 (in head) then count on 5
(“8 … 9, 10, 11, 12, 13”)
Or “8 + 2 = 10…
10 + 3 = 13).</td>
</tr>
</table>

<table>
<tr>
<td colspan="2">


The next step is to bridge through a multiple of 10. The models and images show different ways to find the total or balance the equation.

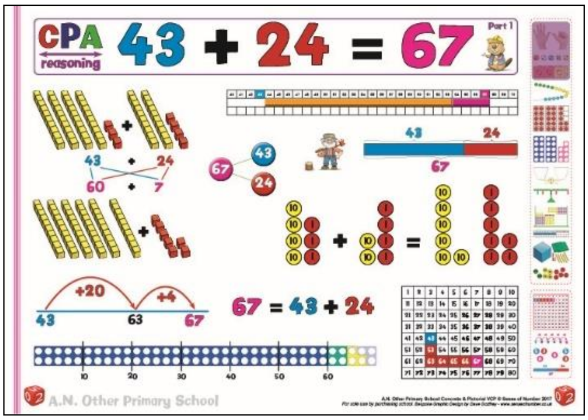
100 square & **number rods** show **counting in 1s**

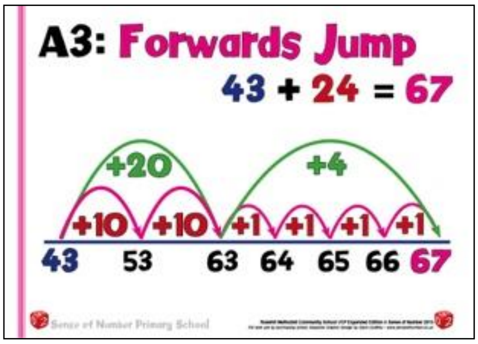
Tens Frames show a **balanced equation** created by passing some counters from the 6 to the 57.

Base 10 is used to show **partitioning**. The 1s have been set out Tens Frame style to visualise how the 5s can be combined to make a 10.
</td>
<td>


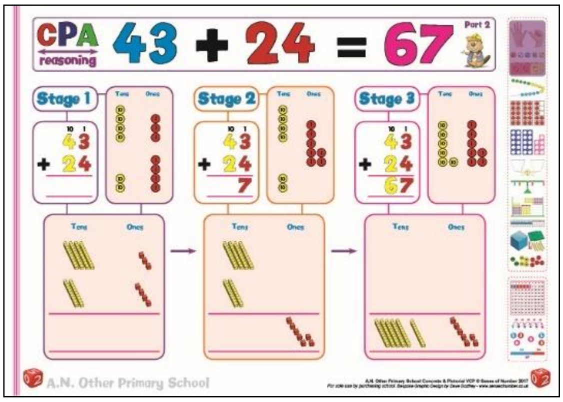
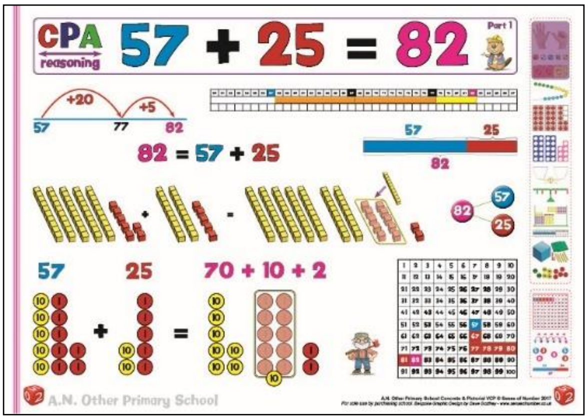
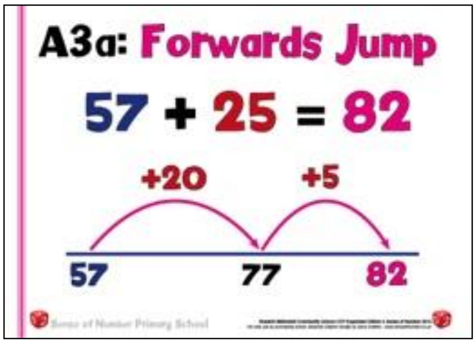
Again, the **number line jotting** can display **counting on**
(57+1+1+1+1+1+1) or
partitioning (57 + 3 + 3)
This picture then becomes the mental strategies: -
57 (in head) - count on 6
("57, 58,59,60,61,62,63")
Or
"**57 + 3 = 60 … 60 + 3 = 63)**
</td>
</tr>
<tr>
<td>

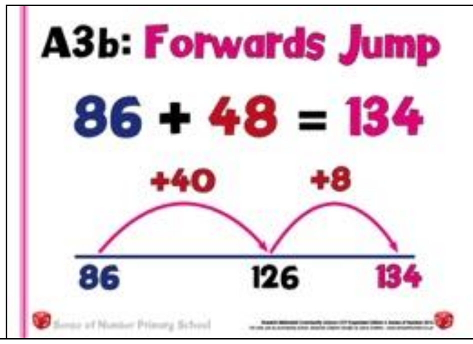
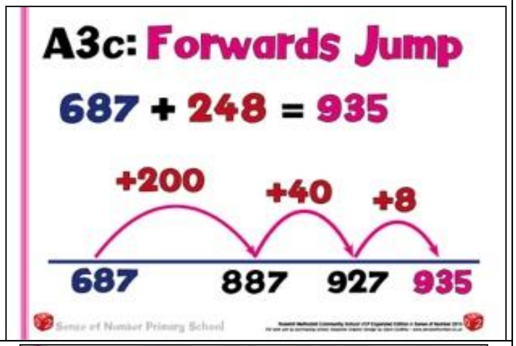
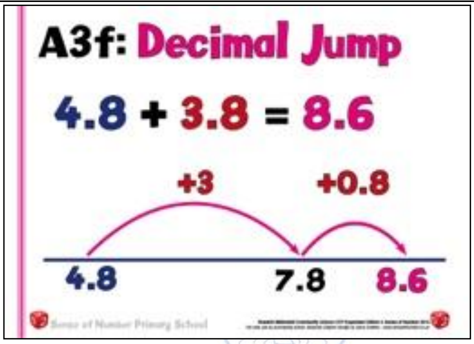
Y2 / 3

</td>
<td>


There are a wide range of ways for children to explore / visualize / create 2-digit calculations.
The **Base 10** and **Place Value Counters** demonstrate **partitioning** and **recombining** (adding 10s and adding 1s)
Numicon shows the **total** once the 10s & 1s have been combined together
The **number rods** and **100 square** display **counting on** 20 then 4.
There is also a **part/whole bar model** and **number bond diagram.**
</td>
<td>


The **number line** shows how to keep one number whole, whilst **partitioning** the other number.
Firstly, add the tens then the ones individually
(43 + 24 =
43+10+10+1+1+1+1)
before counting on in tens and ones
(**43 + 20 = 63 ..**
63 + 4 = 67)
</td>
</tr>
</table>

	 The most crucial element of teaching addition is to ensure that the children have a 'picture' of **partitioning** and **exchanging / regrouping,** meaning that they can visualize the calculation before moving to written / column procedures. This needs to be done practically many times before showing the 'method' in a column. The example above shows how both **Base 10** and **Place Value Counters** can be used to introduce the principle of addition in a column. The numbers themselves (43+24) would usually be added mentally, but you would always introduce the column method process with simple numbers. See '**column method'** section for further examples of column addition using concrete materials to embed understanding.	
	The next stage is to develop both methods to crossing the 10s, then 100s boundary	
	 Even as the calculations increase in value, and start to deal with exchanging, it is important that they are initially visualised via a range of resources / images. Using either Base 10 or place value counters, calculations which involve exchanging can be demonstrated. Note the layout of the resources in Tens Frame style so that it is much easier to see	 By this stage the **number line** no longer counts in individual 10s & 1s. **57 + 20 = 77…** **77 + 5 = 82** **86 + 40 = 126…**

	the five 1s from the 57 added to the five 1s from the 25 to make an extra 10.	**126 + 8 = 134** 
Y3/4	For some children, the number line method can still be used for 3-digit calculations. **687 + 200 = 887…** **887 + 40 = 927…** **927 + 8 = 935** **Or 687 + 200 + 40 + 8 = 935**	
Y5/6	 In Years 5 and 6, if necessary, children can return to the number line method to support their understanding of decimal calculation **4.8 + 3 = 7.8** **7.8 + 0.8 = 8.6** **Or 4.8 + 3 + 0.8 = 8.6** Using number lines support children's thinking if they find partitioning / column addition difficult, as it simply involves counting on in 100s, 10s & 1s. Hopefully, with the first calculation, many children Round & Adjust (4.8 + 4 – 0.2 = 8.6)	

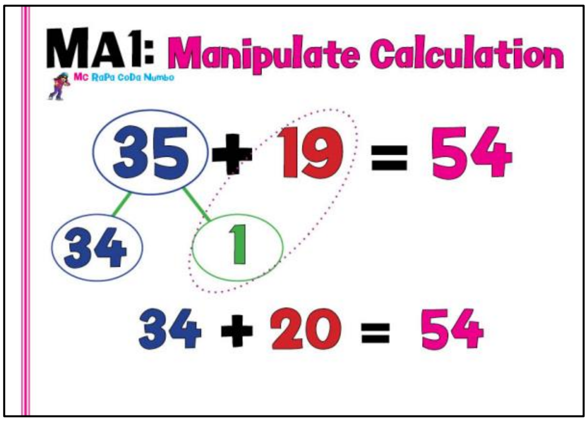
The Part / Whole Method

As part of the 'mastery' approach to the teaching of calculation, especially addition, there are certain methods which begin visually, but which then become strategies that can either be applied mentally or as written jottings.

We have already seen the **Number Line** method appear in both the mental (**Count On**) and written sections of this document. The **Partitioning** method, too, as we will see later, also appears in both sections.

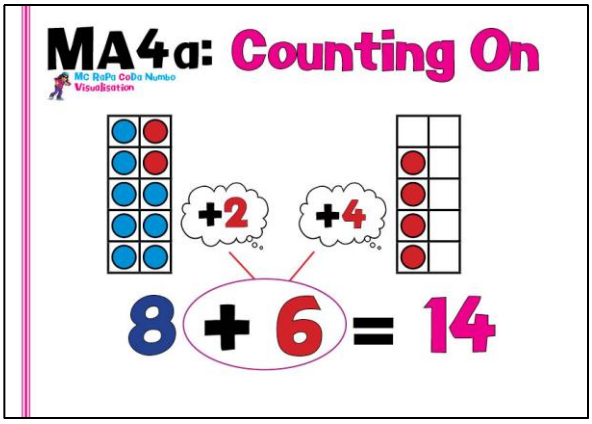
There is, however, an additional method, similar to the mental strategy '**Manipulate The Calculation'**, which falls somewhere between mental and written methods.

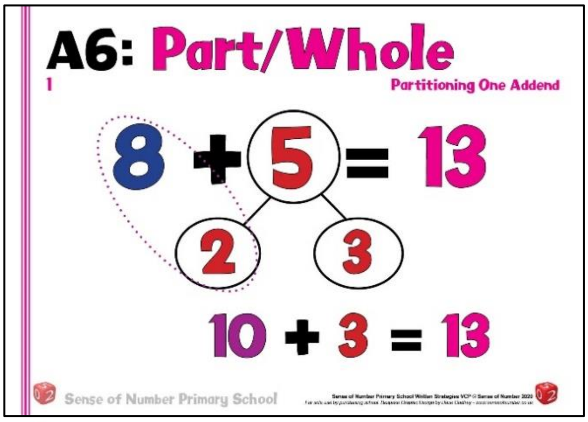
This is known as the '**Part-Whole'** method, and involves **partitioning** just one of the **addends** in order to create a much more simple calculation.



In ‘Manipulate The Calculation’, we have previously seen how **35 + 19** can be simplified by partitioning **35** into **34 + 1**, then adding the ‘**1’** to ‘**19**’ to create a new calculation of **34 + 20.**

The ‘Part-Whole’ method works in exactly the same way.



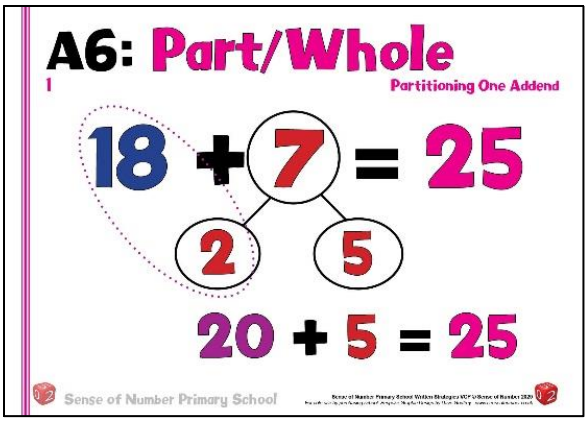


As can be seen from the Tens Frames, the calculation 8 + 5 can be simplified.

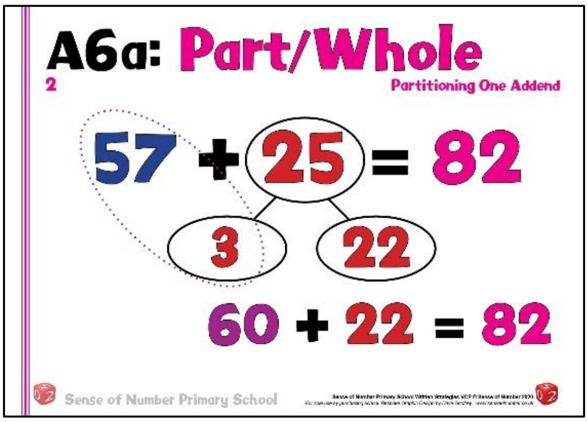
By partitioning the 5 counters in the 2nd frame into ‘2’ and ‘3’, and then ‘passing over’ the ‘2’ to the ‘8’, we can create a new calculation of 10 +3.

The poster on the left shows how this can be recorded as a jotting, using the part-whole model.

Y1



Y2



In Key

Stage 1, the Part-Whole method can be used to simplify calculations when adding both single digit and 2-digit numbers to a 2-digit number.

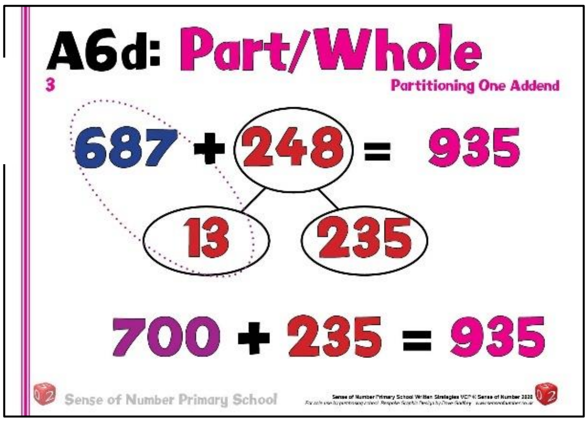
In the examples above, for **18 + 7**, the ‘**7’** has been partitioned into ‘**2’** and ‘**5’** to create **20 + 5.**

Alternatively, the ‘**18’** could have been partitioned into ‘**15’** and **‘3’** to create **15 + 10.**

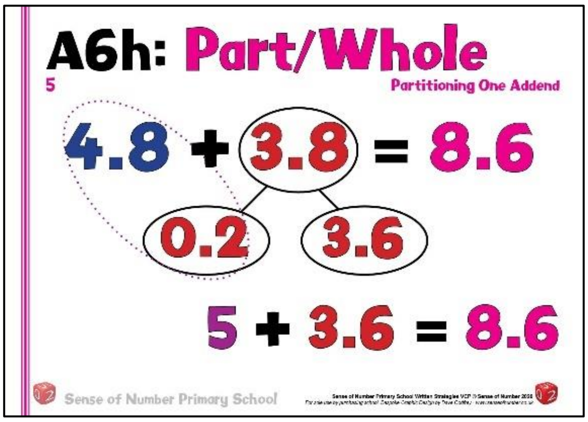
For **57 + 25**, the **‘25’** has been partitioned into **‘3’** and **‘25’** to create **60 + 22.**

Alternatively, the ‘**57’** could have been partitioned into ‘**52’** and **‘5’** to create **52 + 30.**

Y3



Y5



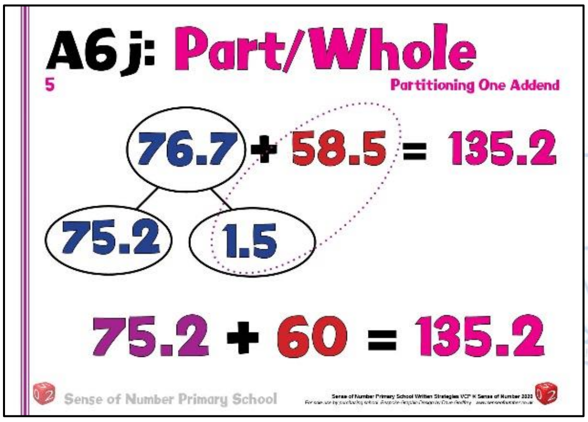
In Key Stage 2, the same strategy can be used to simplify much more complex calculations. It can be applied to 3-digit numbers and also to decimals.

For **687 + 248**, the '**248'** has been partitioned into '**13'** and '**235'** to create **700 + 235.**

Alternatively, the '**687'** could have been partitioned into '**635'** and **'52'** to create **635 + 300.**

For **4.8 + 3.8**, the **'3.8'** has been partitioned into **'0.2'** and **'3.6'** to create **5 + 3.6.**

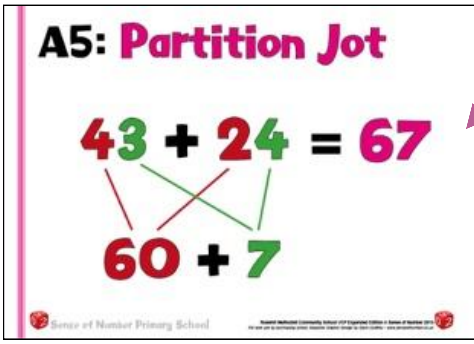
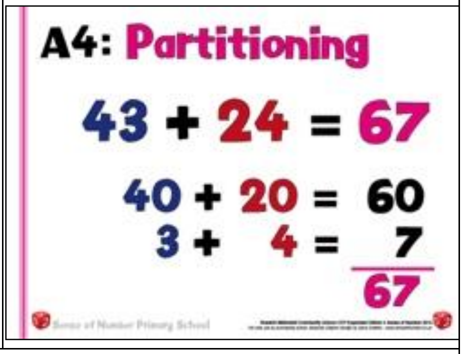
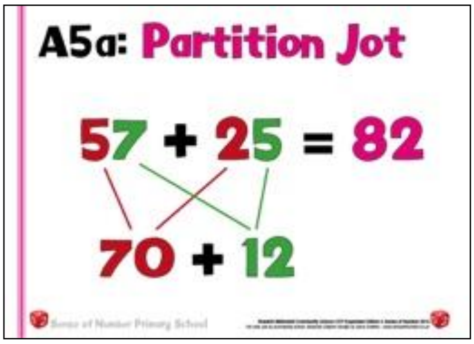
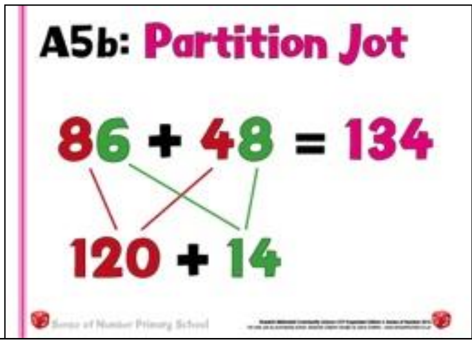
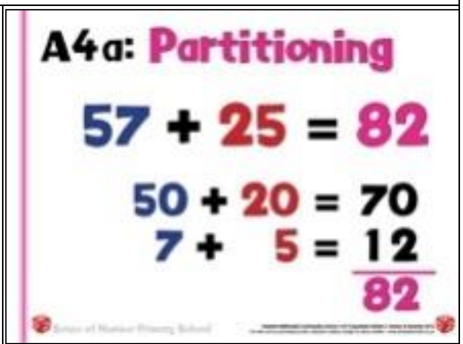
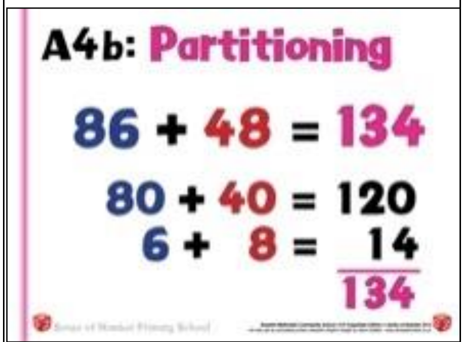
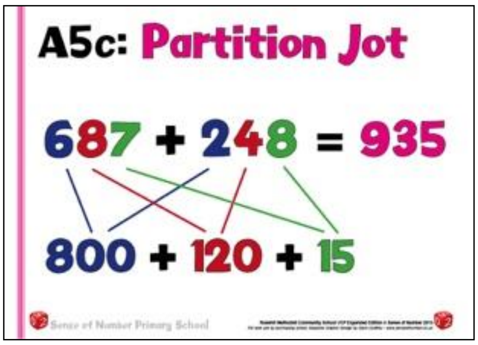
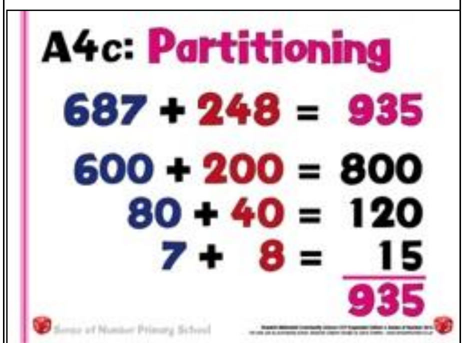
Alternatively, the '**4.8'** could have been partitioned into '**4.6'** and **'0.2'** to create **4.6 + 4.**

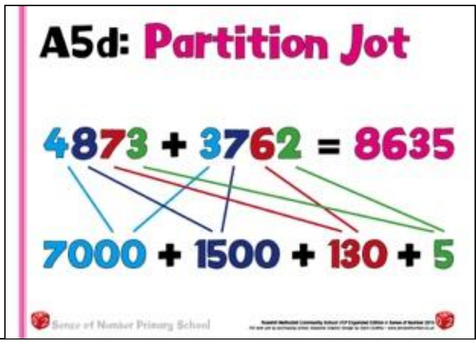
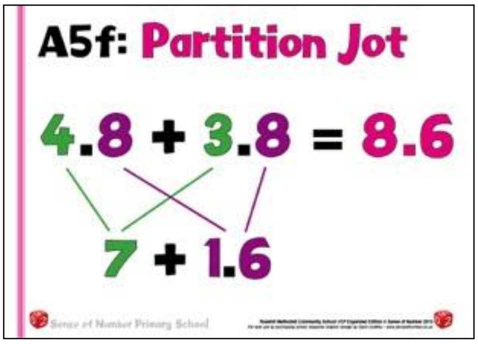
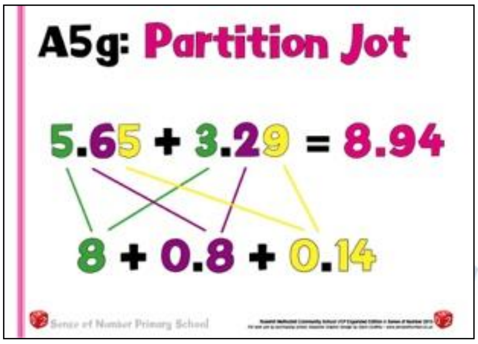
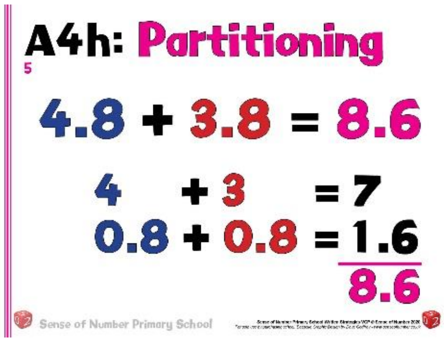
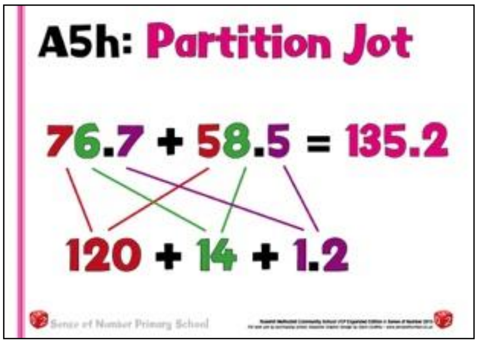
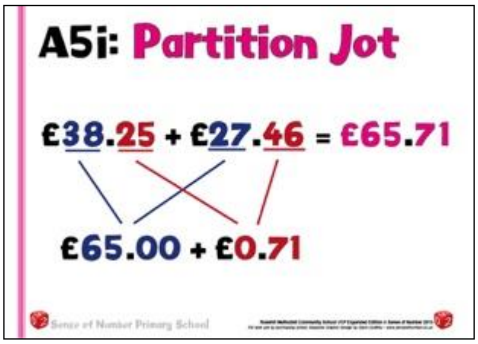


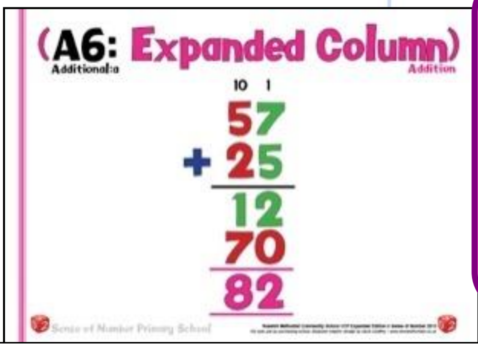
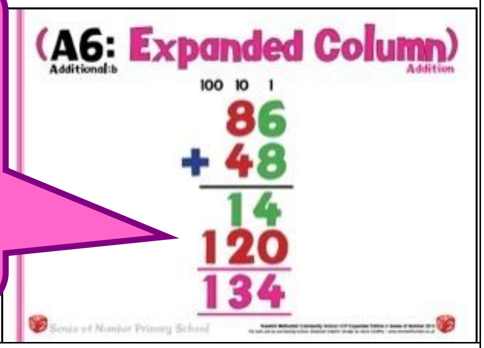
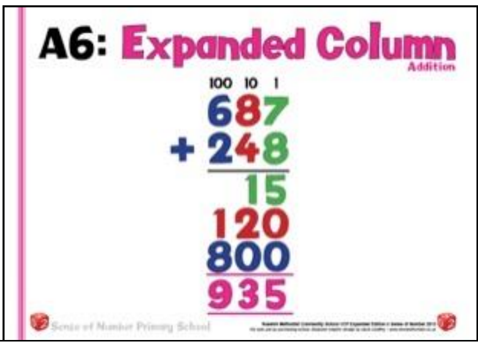
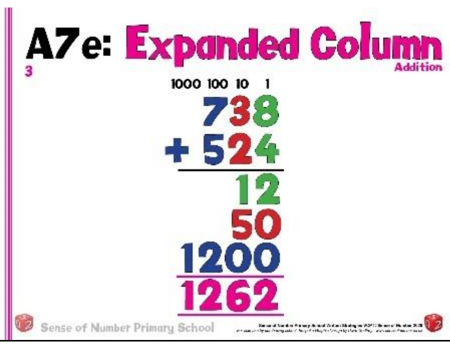
The Part / Whole Model can even simplify more complicated decimal calculations.

The example shows how **76.7 + 58.5**, which children would usually tackle with a column method, could actually be simplified quite easily if the **76.7** was partitioned into **75.2** and **1.5.**

The resulting calculation would be **75.2 + 60**

Stage 2	Partition Jot	Alternative Method: Traditional Partitioning
Y2/3	**Traditionally, partitioning has been presented using the method on the right. Although this does support place value and the use of arrow cards, it is very laborious, so it is suggested that adopting the 'partition jot' method will improve speed and consistency for mental to written (or written to mental) progression**	Record steps in addition using partition, initially as a jotting: - **43 + 24 =** 40 + 20 + 3 + 4 = 60 + 7 = 67 Or, preferably
	As soon as possible, refine this method to a much quicker and clearer '**Partition Jot'** approach 	
	As before, develop these methods, especially Partition Jot, towards crossing the 10s and then 100s**.**	
	 	 
Y3/4	 This method will soon become the recognised jotting to support the teaching of partitioning	

	. Partition jot can be easily extended to 3 and even 4-digit numbers when appropriate.  	For certain children, the traditional partitioning method can still be used for 3-digit numbers, but it is probably too laborious for 4-digit numbers.
Y5/6	  Partition jot is also extremely effective as a quicker alternative to column addition for decimals.	 Some simple decimal calculations can also be completed this way.
	  For children with higher-level decimal place value skills, partition jot can be used with more complex decimal calculations or money.	

Stage 3	Expanded Method in Columns
Y3	**Column methods of addition are introduced in Year 3, but it is crucial that they still see mental calculation as their first principle, especially for 2-digit numbers.** **Column methods should only be used for more difficult calculations, usually with 3-digit numbers that cross the Thousands boundary or most calculations involving 4-digit numbers and above.** **N.B. Even when dealing with bigger numbers / decimals, children should still look for the opportunity to calculate mentally (E.g. 4675 + 1998)**
	2 digit examples are used below simply to introduce column methods to the children. Most children would continue to answer these calculations mentally or using a simple jotting.
	Using columns, children need to learn the principle of adding Ones first rather than Tens.
	The 'expanded' method is a very effective introduction to column addition. It continues to use the partitioning strategy that the children are already familiar with, but begins to set out calculations vertically. It is particularly helpful for automatically 'dealing' with the 'carry' digit. **It is crucial, though, to ensure practical apparatus is used first before any sort of column procedure is introduced (see egs below in the 'Column Method' section.**
Y3/4	**A. Single 'carry' in Ones**  **'80 plus 40 equals 120 , because '8 Tens plus 4 Tens equals 12 Tens.** **B. 'Carry' in Ones and Tens** 
	Once this method is understood, it can quickly be adapted to using with 3-digit numbers. It is rarely used for 4 digits and beyond as it becomes too unwieldy.
Y3/4	**C. 'Carry' in Ones and Tens**  **C. 'Carry' in Ones and Hundreds** 
	The time spent on practicing the expanded method will depend on security of number facts recall and understanding of place value. Once the children have had enough experience in using expanded addition and have also used practical resources (Base 10 / place value counters) to model exchanging in columns, they can be taken on to standard, 'traditional' column addition.

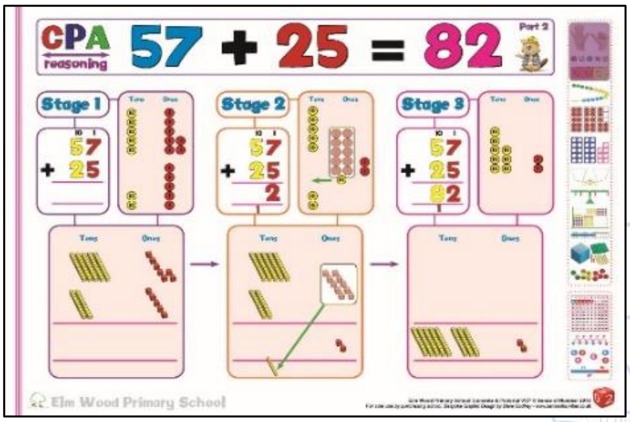
Stage 4	Column Method
Y3/4	As with the expanded method, begin with 2-digit numbers, simply to demonstrate the method, before moving to 3-digit numbers. **Make it <u>very clear</u> to the children that they are still expected to deal with all 2-digit (and many 3 digit) calculations mentally (or with a jotting),** and that the column method is designed for numbers that are too difficult to access using these ways. The column procedure **<u>is not</u>** intended for use with 2-digit numbers unless completely necessary for certain children who have no other strategy.

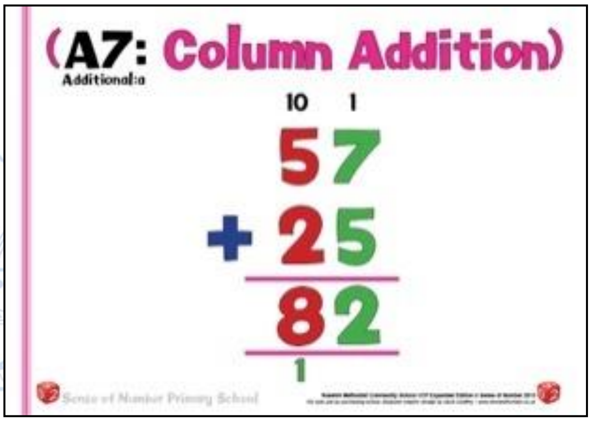
'Carry' Ones then Ones and Tens

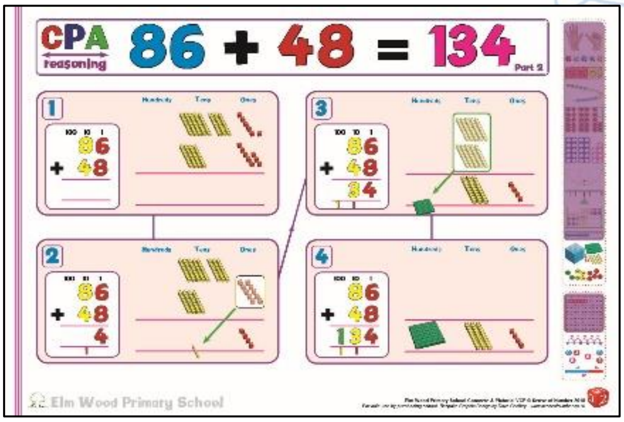
The most important part of the transition from mental and jotting approaches towards the use of column methods is the initial use of concrete materials to make / create and discuss the understanding behind the method. If children have had regular experience of using **Base 10 apparatus** and then **Place Value Counters** to make and solve calculations,

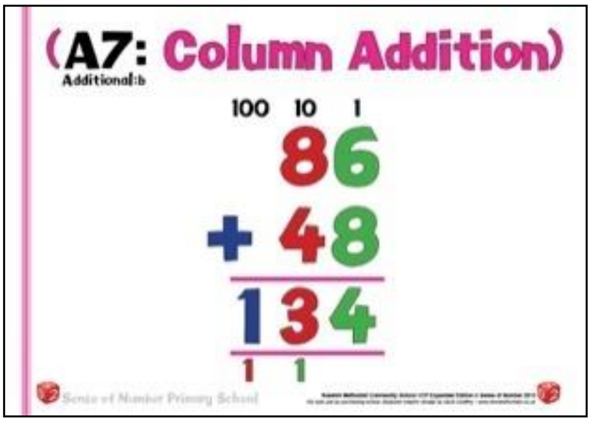
(especially the regrouping of **10 'Ones' into 1 'Ten'** and **10 'Tens' into 1 'Hundred'**)

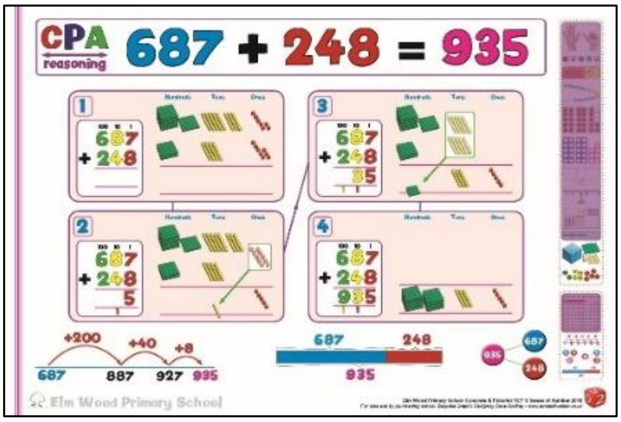
then the column method is simply a written version of what they already understand.

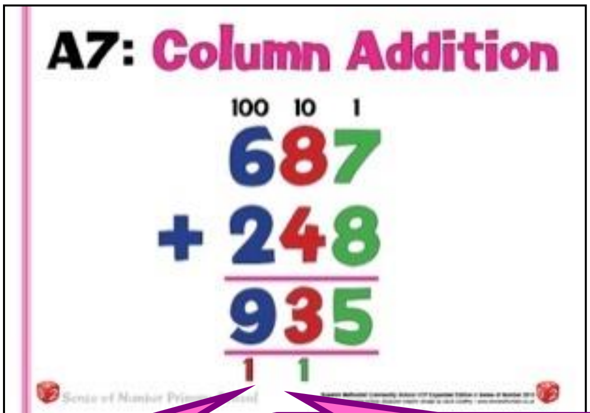












Record carry digits below the line.

Use the words 'carry ten' and 'carry hundred', not 'carry one'

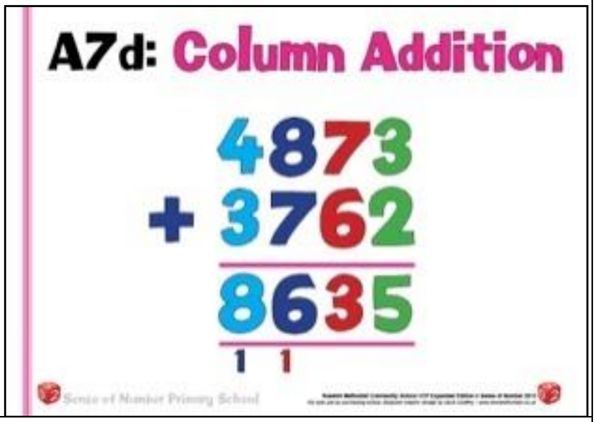
Allow the children plenty of opportunity to make both 2 and especially 3 digit calculations with manipulatives, in particular Base 10. Make sure that the children are encouraged to always lay out apparatus '**5 Style'** so that it is much easier to read the calculation.

For example, in **57 + 25,** take note of the layout of the Ones with both the Base 10 and / or the Place Value Counters
This allows the children to see **7 Ones** (as **5 Ones +2 Ones**) to be added to **5 Ones.**
Immediately, both sets of **5 Ones** can then be regrouped into **10 Ones** then exchanged for **1 Ten**

In **687 + 248,** the layout of the apparatus means that it is really easy to recognise the **6 Hundreds** (**500 + 100**), the **8 Tens** (**50 + 30**) and the **7 Ones** (**5 + 2**).
This, in turn, means that the children can regroup both the **10 Ones** into **1 Ten** and then the **10 Tens** into **1 Hundred** straight away.

Y4

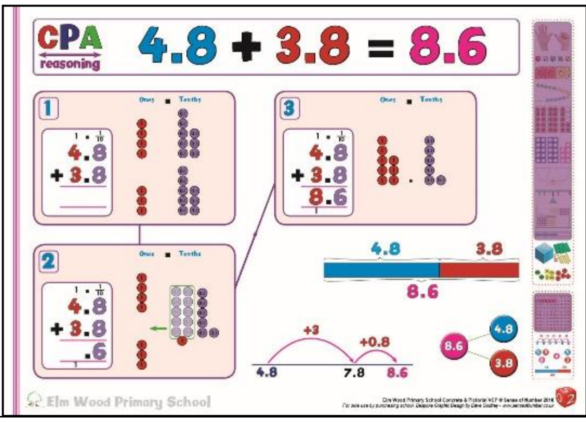
Once confident with place value, regrouping & exchanging, and being able to represent and explain any 3 digit column method using apparatus, the column method can now be introduced for use with 4-digit numbers.
At this point the apparatus is not necessary as the calculation procedure should be fully understood.

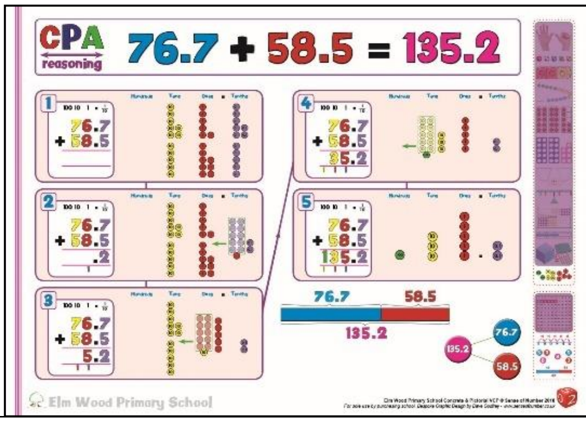


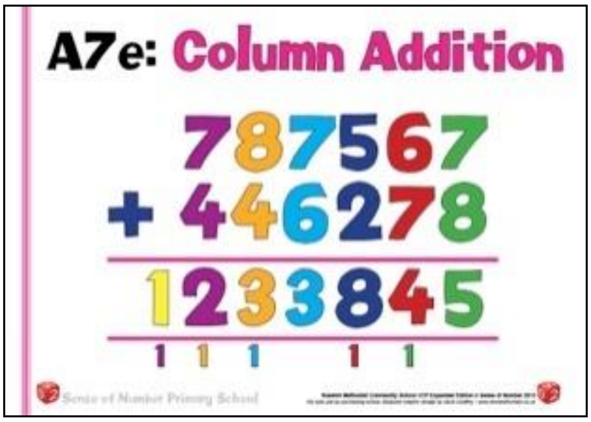
Y5/6

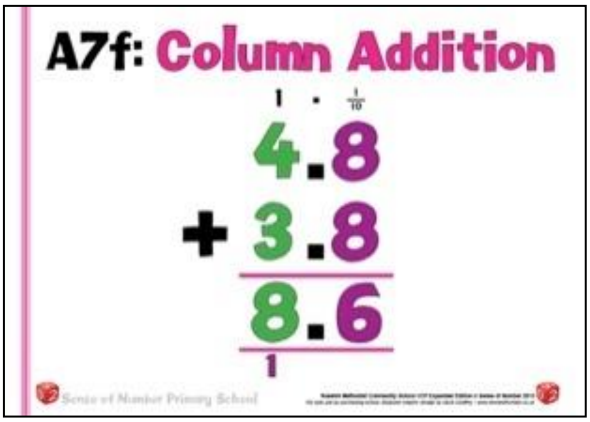
In Years 5&6, extend column methods to calculations involving 5 & 6 digit numbers, and then to a wide range of decimal calculations

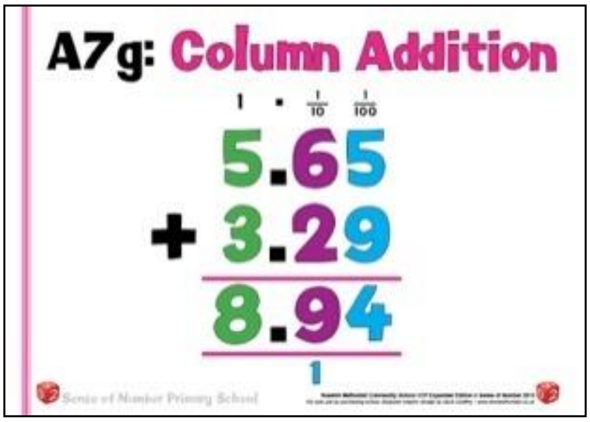
Even with decimals, it makes sense to embed the CPA approach first, where children are encouraged to use Decimal Place Value Counters to create and practice the calculation by making it completely visual. This will also involve explaining the place value principles behind the calculation. They will need to describe the method and explore what occurs when the children are exchanging / regrouping tenths and hundredths

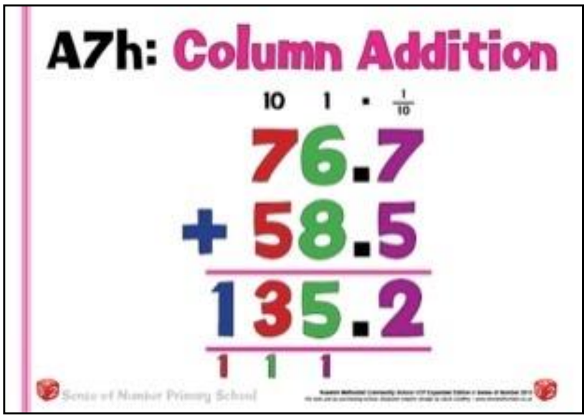






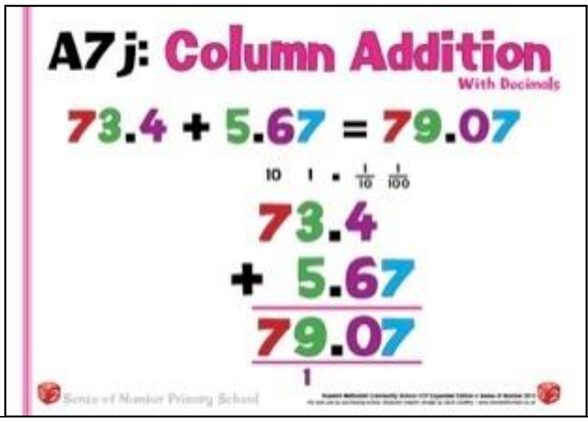




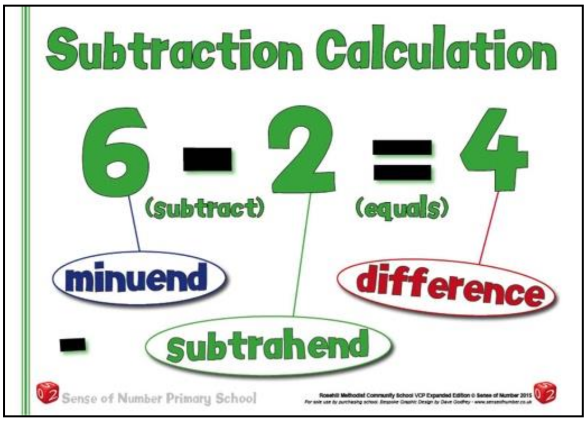


If children make repeated errors at any stage of column addition , they can return to the expanded method, an earlier jotting or to the use of manipulatives such as Base 10 or Place Value Counters.

The key skill in upper Key Stage 2 that needs to then be developed, usually in Year 6, is the laying out of the column method for calculations with decimals in different places.



Subtraction Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- **recall all addition and subtraction facts to 20;**
- **subtract multiples of 10 (such as 160 – 70) using the related subtraction fact (e.g. 16 – 7), and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Reading Subtraction Calculations / Common Misconceptions

It is fairly common for children (and teachers) to view every subtraction in one simplistic way – as a '*take away*'. Even when reading subtraction calculations such as **42 – 38** the most common way in which it is read aloud would be *'42 take away 38'*.

This way of reading subtraction calculations can cause many problems, both in understanding what subtraction actually means, and also in choosing the best method of actually completing the calculation.

Using the example cited, 42 – 38, although we could count back 30 then count back 8 from 42 (with or without a number line), and might even count out 42 items then remove 38 of them (Base 10 or real life objects), the easiest / most efficient way to solve the calculation would be to 'count on' 4 from 38 to 42 (or simply to recognise the difference of 4). If we read the calculation as 'take away' then we are implying that this is the strategy that should be chosen.

Therefore, it is vital that everybody, both teachers and children, are encouraged / persuaded to read all calculations using the correct language (***'42 subtract 38'***, ***'42 minus 38'*** or '***What is the difference between 42 and 38?***') and then allow the person answering the question the opportunity to choose the most appropriate strategy (**Count on** or **count back)**

As can be seen in the later parts of this section, the **'count on'** approach is usually demonstrated via the use of a **number line or jotting,** whilst **the 'removing items / take away '** approach is explored using **Real Life Objects**, **Generic Concrete Materials, Base 10 apparatus** and **Place Value Counters** (as with addition) to allow visualisation and understanding of the exchanging / regrouping principles.

<u>NOTE:</u> Children should always look at the actual numbers each time they see a calculation and decide whether or not their favoured method is most appropriate (e.g. If there are zeroes in a calculation such as 406 -178) then the '**counting on'** approach may well be the best method in that particular instance), even if they would normally use a column procedure.

Therefore, when subtracting, whether mental or written, children will mainly choose between two main **strategies** to find the difference between two numbers: -

Counting Back ***OR*** ***Counting On***

When should we count back and when should we count on?

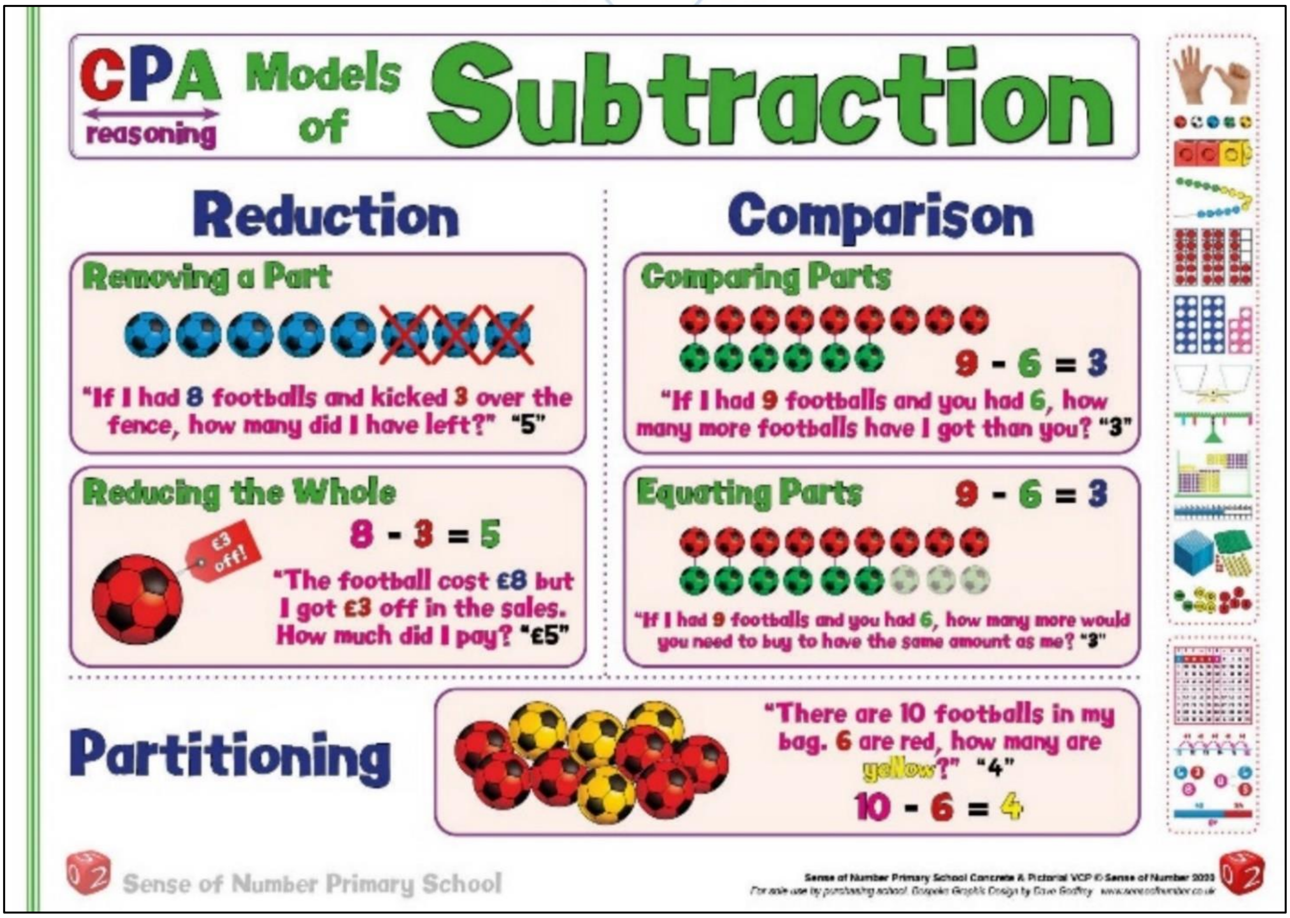
This will alter depending on the calculation (see below), but often the following rules appl**y;**

If the numbers are far apart, or there isn't much to subtract (278 – 24) then count back.

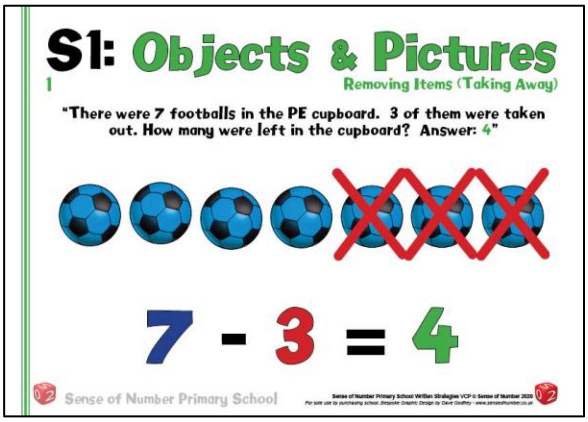
In many cases, either strategy would be suitable, depending on preference (743 – 476)

If the numbers are close together (206 – 188), then count up

<u>Models of Subtraction</u>



The poster on the previous page gives a clear overview of subtraction and allows any member of teaching staff (especially the Subject Leader) to realise that there are actually **3 key structures (and potentially 5 ways)** to visualise subtraction, not just one.



Two of the ways (on the left hand side of the poster) involve the structure of **reduction.**

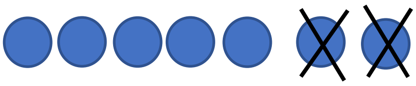
The first of these is often referred to **as 'removing items' or 'taking away'**

I have 7 cakes. I eat 3 of them, how many are left?

There were 7 children in the class but 3 of them went outside. How many remained?

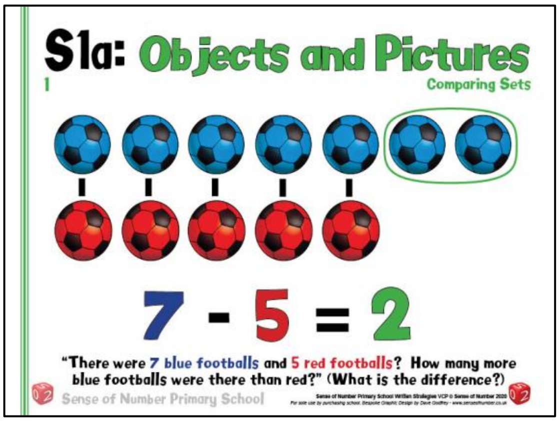
or reducing the whole

The price of the bag of cakes cost £7. At the end of the day it was reduced by £3. How much did I pay?).



In both of these examples we end up with 'less' than we started with, which fit in with the standardised view of subtraction as 'take away' and 'less than'

The right-hand side of the column explores the other key aspect of subtraction, that of comparison.



In a standard 'comparison' story

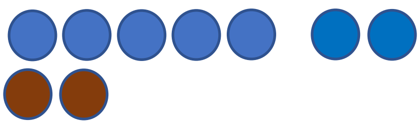
(***I have 7 cakes and my friend has 2 cakes. How many more cakes do I have?)***

or an inverse of addition story

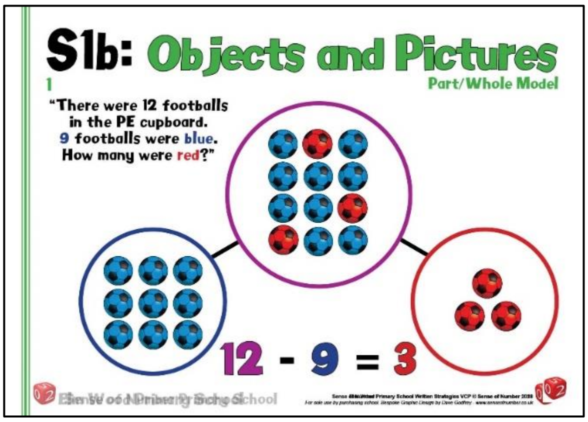
(***It costs £7 to buy the cakes, but I only have £2. How much more do I need?)*** there is no taking away whatsoever.

In the first example there are actually 9 cakes altogether, none of them are eaten / taken away; they are simply compared.

In the second example, there is only £2 and £5 more is needed. Despite the fact that nothing is removed, these are both clearly subtraction calculations.



The final example is neither removing items or comparing. It is a way of using subtraction to discuss the parts of a whole.



In a ‘part / whole’ story

There are 7 cakes in a bag, either cream cakes or chocolate brownies. 2 of them are brownies, how many are cream cakes?) nothing is taken away and nothing is added or compared. Subtraction (as with a bar model or number bond diagram) explores the two parts. If there are 7 in total and 2 of them are brownies, then 7 subtract 2 must give me the number of cream cakes.

As with addition, it is important that children are introduced to the different models for subtraction (especially ‘take away’ and ‘compare’ using practical resources.

In EYFS this would be real objects such as footballs, shells, cakes, cars, dinosaurs etc.

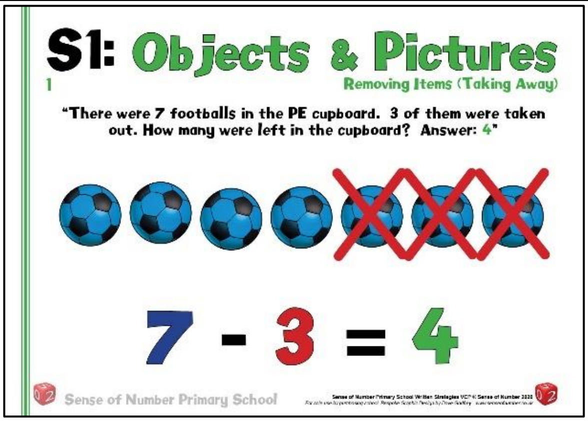
In KS1 this would progress from concrete materials such as counters or cubes to pictorial models and sketches.

The two main models of subtraction – ‘removing items’ and ‘comparison’, will develop into the two main mental / written methods of ‘subtraction by counting back’ (take away’ / decomposition) and ‘subtraction by counting on’ (‘complementary addition’ / counting up on a number line).

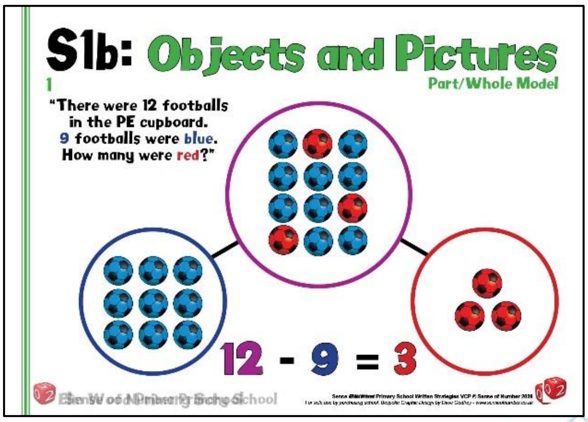
All of these principles are explored in the actual written calculation policy below

Written Methods of Subtraction

INTRO	Subtraction by counting back (or reduction)	Subtraction by counting up (or complementary addition)
FS/Y1	Early subtraction in EYFS will primarily be concerned with reduction, and will be modelled using a wide range of models and resources. These will usually be natural resources and real-life objects, and will often be part of a story telling scenario where the children can ‘make’ subtraction to tell the story	



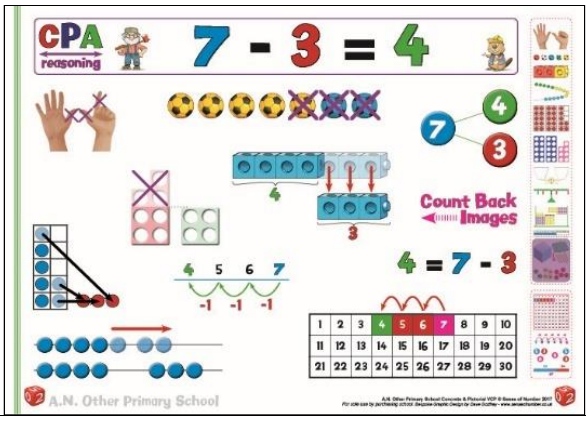
In Year 1, children will learn and visualize both the reduction and



partitioning (part / whole) models of subtraction

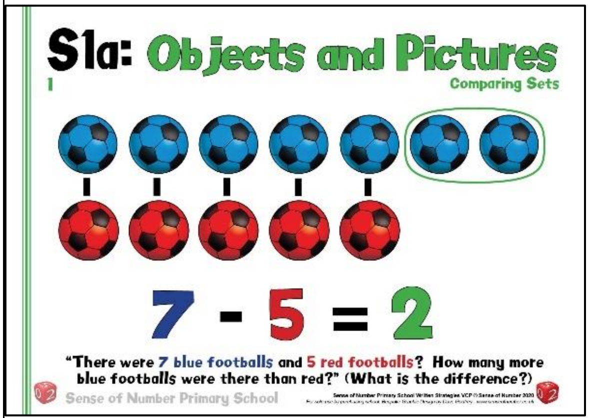
Using resources such as multi-link cubes, Numicon, Ten Frames, bead strings and pictures (like the footballs shown below) the reduction and partitioning approaches will be modelled.

Once these are secure, images such as the desktop number track / line can be used to practice practical subtraction, then developed into counting back on demarcated number lines.



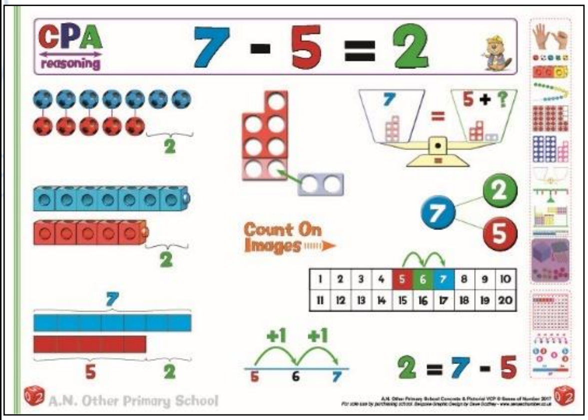
In Year 1, it is also vital that children understand the concept of subtraction as **'**finding a difference' by **comparison** and realise that **<u>any</u>** subtraction can be answered in 2 different ways, either by counting up or counting back.

Again, this needs to be modelled and



consolidated regularly using a wide range of resources, especially multilink towers, counters and Numicon.

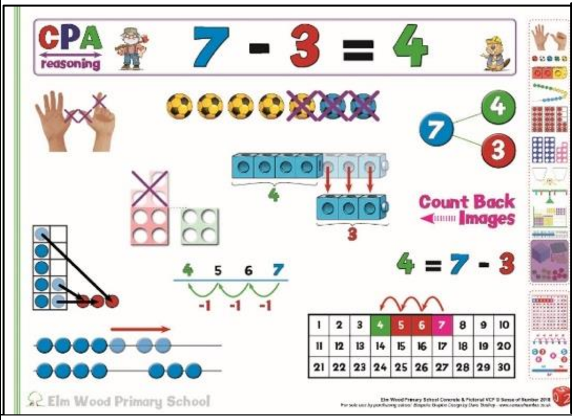
The images below also show the early version of bar modelling, as well as the Numicon pieces being used to demonstrate the equals sign as a balance (7 is equal to 5 + __



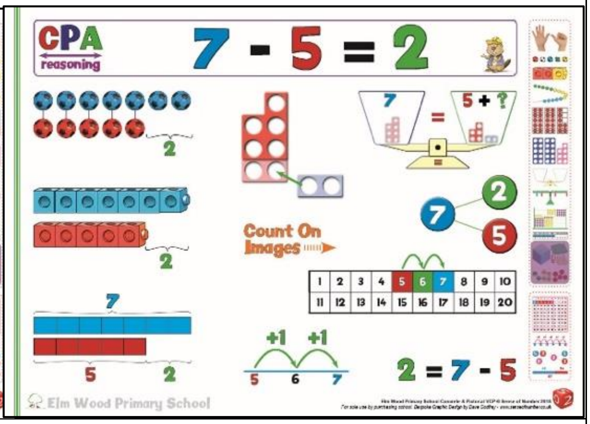
Stage 1	Using the empty number line	
	Subtraction by counting back (or taking away)	**Subtraction by counting up (or complementary addition)**

Before developing any jottings, mental methods or column procedures, children need to explore, discuss and visualize the two main models of subtraction using a wide range of models, images and apparatus. (including Numicon, multi-link cubes and Tens Frames). It is not advised to begin using a number line as a tool for calculation until the children understand the principles behind why it is being used. Otherwise the number line becomes a method, rather than a tool to embed and further understanding and fluency.

Images for 'counting back'



Images for 'counting on

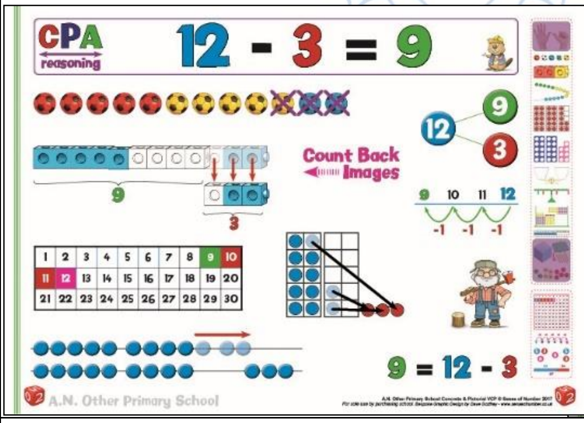


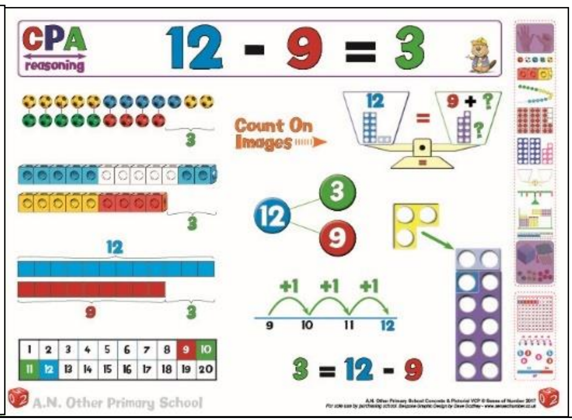
Y1

Before using a number line, therefore, children need to be given the opportunity to 'make' both models of subtraction using resources.

For 12 - 3, the multi-link and Ten Frame pictures clearly show 12-2-1, whilst the footballs display the 'take away'.

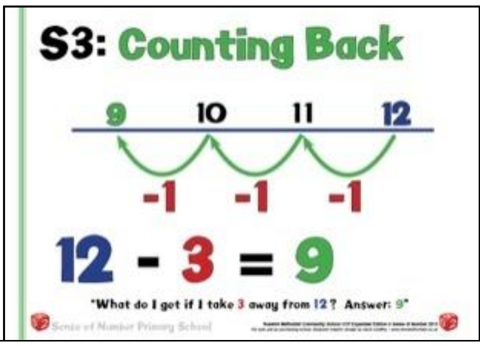
For 12 – 9 the number rods, cubes and footballs display a clear comparison, where the difference of 3 can be seen.



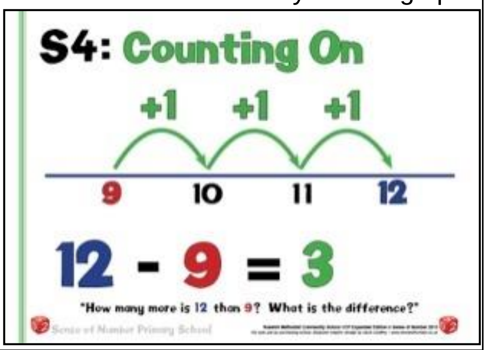


The empty number line helps to record or explain the steps in mental subtraction.
It is an ideal model for **counting back** and **bridging ten**, as the steps can be shown clearly.
It can also show **counting up** from the smaller to the larger number to **find the difference**

Steps often bridge through a multiple of 10.

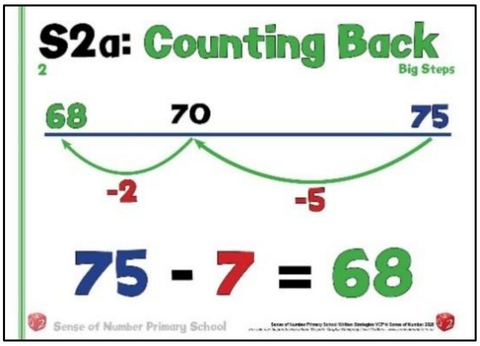


Small differences are found by counting up

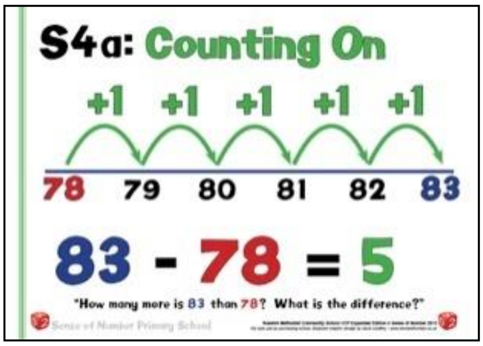


Y2/3

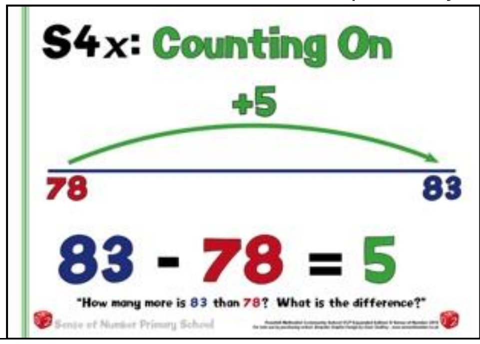
This is developed into crossing any multiple of 10 boundary.

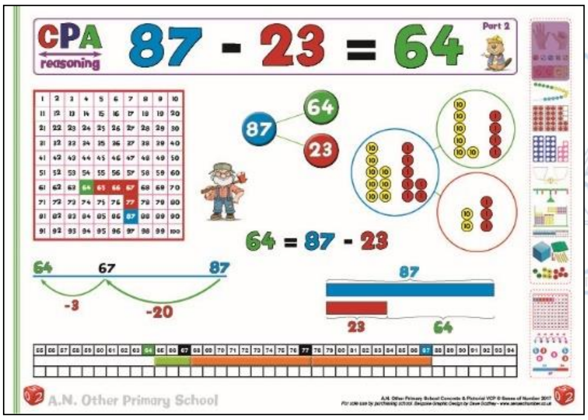


For 2 (or 3) digit numbers close together, count up. First, count in ones



Then, use number facts to count up in one jump



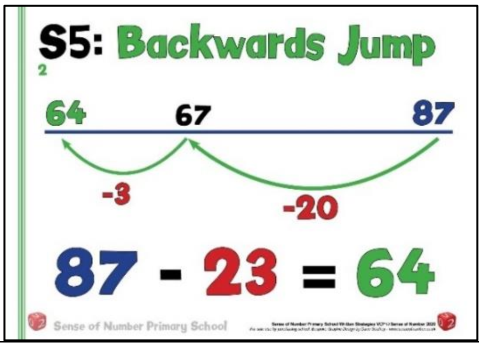


For 2-digit numbers, count back in 10s and 1s. As with addition, it is important to let the children manipulate materials in order to 'see' the calculation.

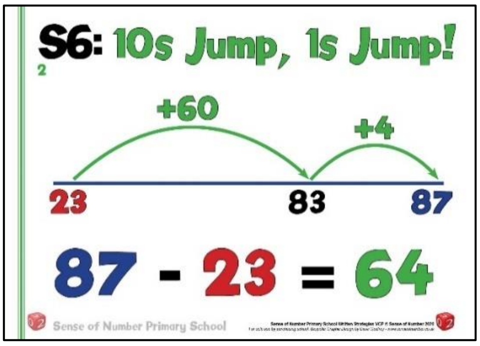
Base 10 and Place Value Counters are particularly crucial as these are images which are used throughout the topic (see expanded method section for these images).

The image above uses Place Value Counters to demonstrate the Number Bond diagram.

Once secure, model the use of the empty number line where tens and ones are subtracted in single jumps **(87 – 20 – 3)**

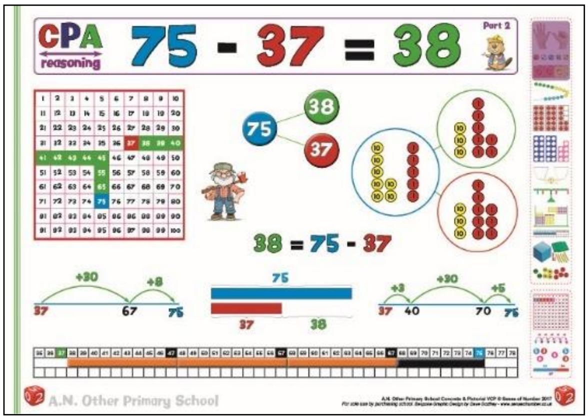


Although this calculation is probably easier to answer by counting back, the 'count on' method is still effective and is equally as efficient



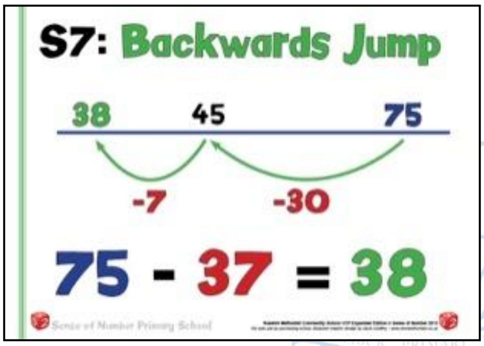
Some numbers (75 – 37) can be subtracted just as quickly either way.

The images below show 75-37 as a Number Bond Diagram (also visualised with Place Value Counters), both as a ‘count up’ image using the number line (explored below in 2 different ways) and number rods, and as a take away / count back image on the 100 Square.

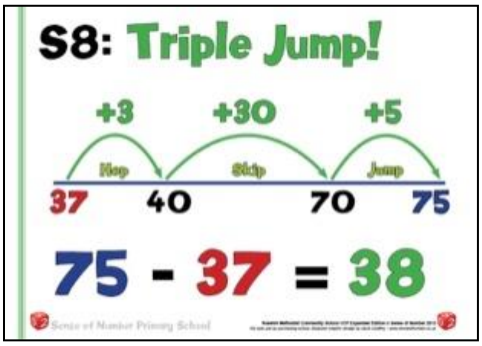


FThe number line itself is an excellent visual jotting for both counting on and counting back, depending on the children’s preferred strategy (see below)

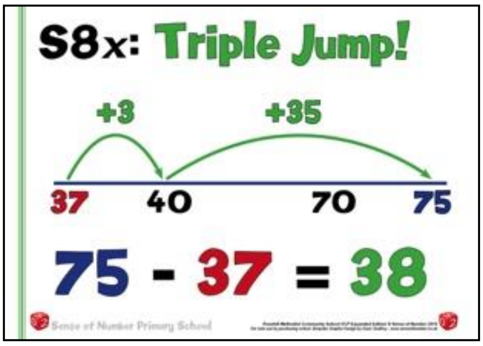
Either count back 30 then count back 7



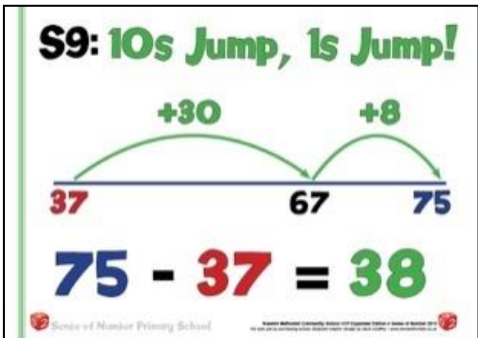
Or count up from smaller to the larger number, initially with a ‘triple jump’ strategy of jumping to the next 10, then multiples of 10, then to the target number.



This can also be done in 2 jumps.



Some children prefer to jump in tens and ones, which is an equally valid strategy, as it links to the mental skill of *‘counting up from any number in ten*



The Part / Whole Method

As part of the 'mastery' approach to the teaching of calculation, there are certain methods which begin visually, but which then become strategies that can be applied either mentally or written.

We have already seen the **Number Line** method appear in both the mental (**Count On & Count Back**) and written sections (**Counting Back, Counting On, Backwards Jump, 10s Jump 1s Jump, Triple Jump**) of this document.

There is, however, an additional method, also part of the mental strategies section of this document, which involves making the calculation easier to access. This strategy, known as the **Part / Whole Method**, is, in effect, an alternative way of partitioning numbers when calculating.

A standard partition (similar to the partitioning method of addition) is generally not an effective method to teach, as it only works when all of the digit values in the minuend are higher than the digit values in the subtrahend.

E.g **87 – 23** could be worked out by partitioning

(**80 – 20** = 60 and **7 – 3** = 4 therefore **87 – 23** = 64

But **83 – 27** could not be worked out the same way without using negative numbers

(**80 – 20 =** 60 but **3 – 7** can't be worked out using positive numbers.

Therefore, in order to **subtract** by **partitioning**, number sense is needed, and an alternative approach needs to be deployed.

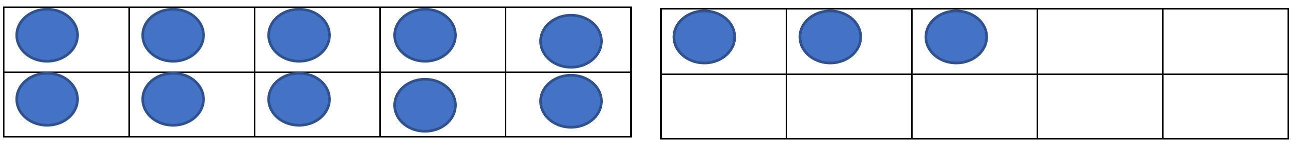
This is known as the '**Part-Whole'** method, and, like addition (where either of the addends can be partitioned), the part-whole method of subtraction allows either the **subtrahend** (in most cases) or the **minuend** (when working at even greater depth) to be partitioned in order to create a much more accessible calculation.

In Key Stage 1, both approaches can be demonstrated with apparatus.

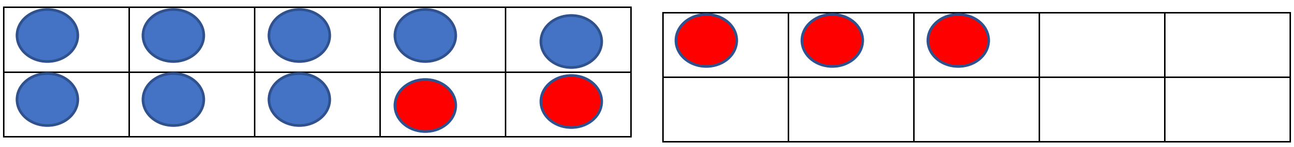
Partitioning The Subtrahend

For the calculation 13 – 5, rather than counting back (or removing) 5, the **subtrahend** can be partitioned into 3 and 2, as the pictures below show. We begin with 13 in 2 Tens Frames (a)

a)



b)



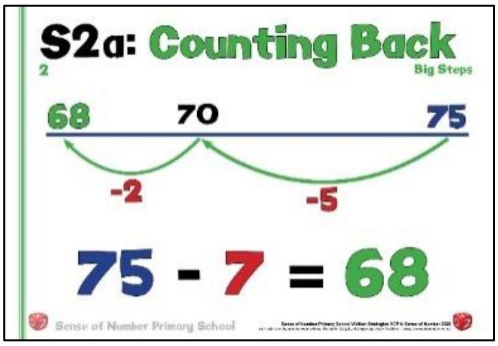
First (b), we subtract 3 from the second Ten Frame, then subtract 2 from the first Ten Frame.

As a part-whole 'jotting', the calculation would be written

13 - 5 = 8

3 2

13 – 3 = 10
10 – 2 = 8



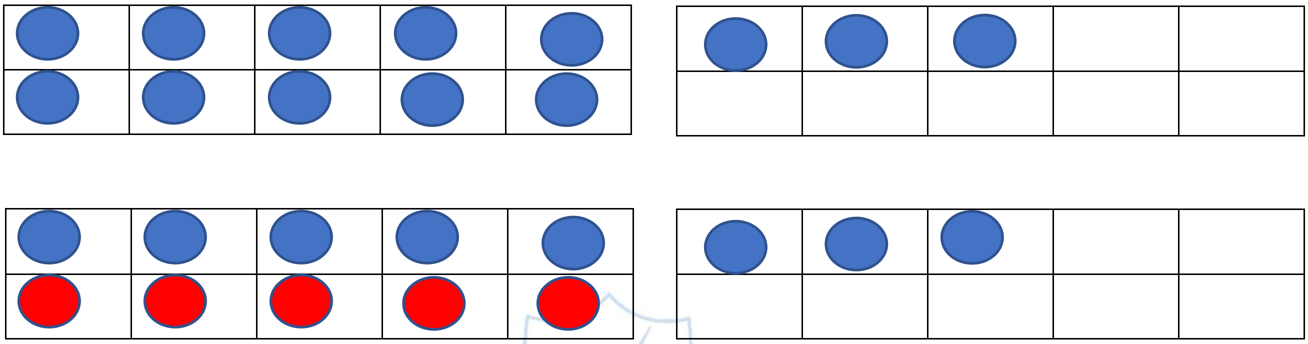
This approach links to the number line method explored earlier for **75 – 7**, where the minuend was partitioned into **5 and 2, and** basically gives an different layout without the need to draw a line.

Partitioning The Minuend

The alternative approach to the Part-Whole method, which generally takes a little longer for the children to use and understand effectively, is to partition the **minuend** instead and **subtract** the **subtrahend** from the **larger part.**

As the pictures show, we again begin with 13 in 2 Tens Frames (a)

a)



In this instance (b), we subtract the 5 from the first Tens frame, and then we clearly see the remaining difference is 8.

As a part-whole 'jotting', the calculation would be written

13 - 5 = 8

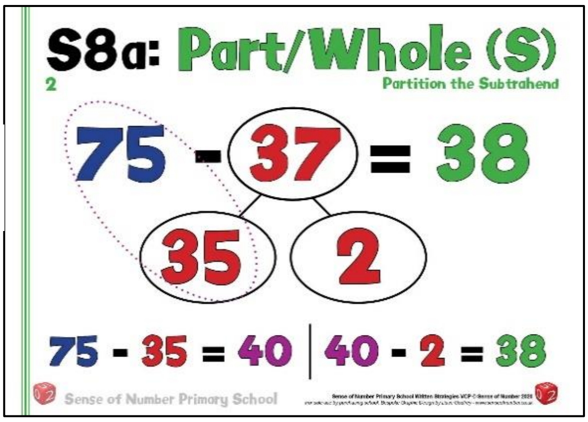
10 3

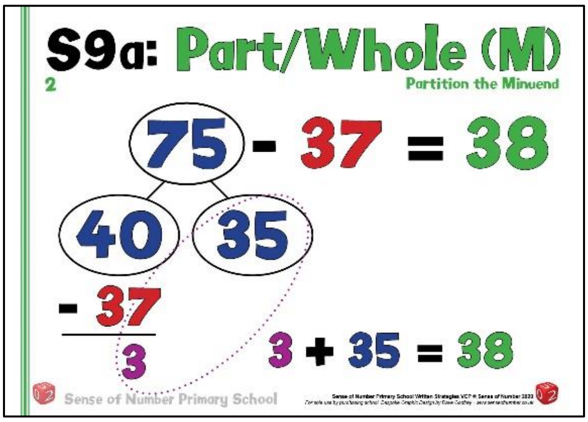
10 – 5 = 5
5 + 3 = 8

The slides below demonstrate how the part-whole method can be developed throughout the whole school, using both approaches.

Partitioning The Subtrahend | **Partitioning The Minuend**

Y2





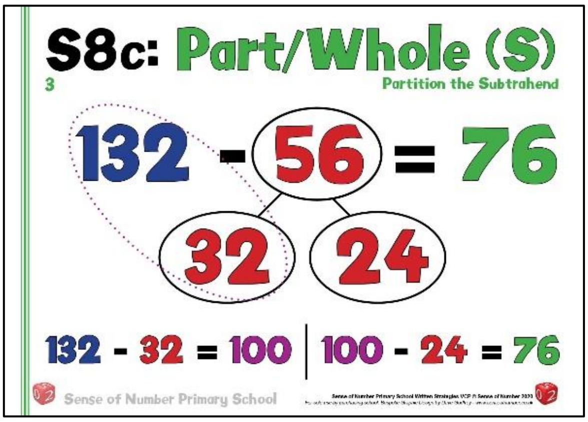
The calculation **75 – 37** can be simplified if the subtrahend of **37** is partitioned into **35 and 2**.
The **35** is subtracted from the **75** (equalling **40**)

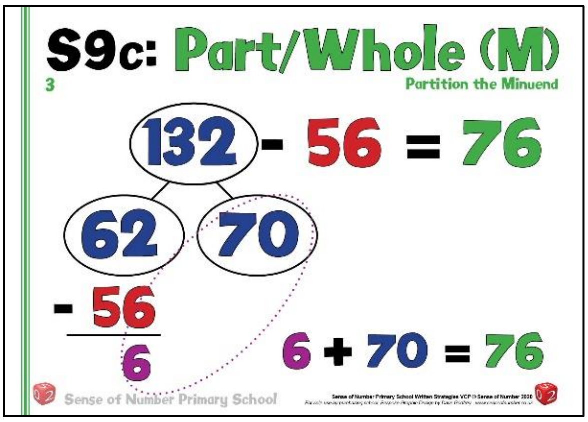
Alternatively, the minuend of **75** can be partitioned into **40 and 35.**
The **37** is then subtracted from the 40

before the **2** is subtracted from the **40**, giving an overall difference of **38**.

(equalling **3**) and the **3** is added to the remaining **35**, also giving an overall difference of **38.**

Y3

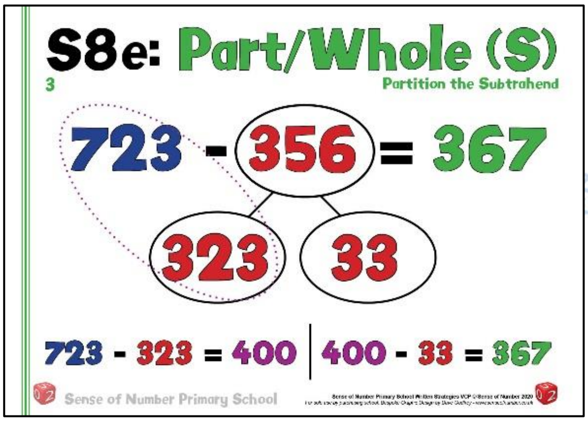


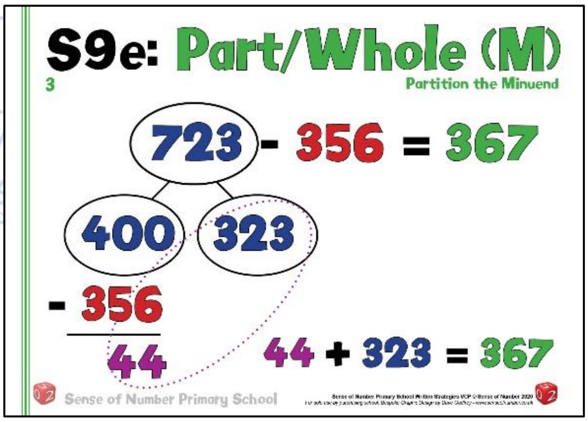


Both methods continue into Key Stage 2, and can be used to subtract either 2 or 3 digit numbers.

In the example above, the subtrahend of **56** is partitioned into **32 and 24** so that the **32** can be simply subtracted from **132**, leaving **100**.
The remaining **24** is then subtracted from **100**, giving a difference of **76.**

Partitioning the minuend of **132**, however, into **62 and 70**, means that the **56** can be subtracted from **62** (equalling **6**)
The **6** is then added to the **70**, again giving an overall difference of **76**.

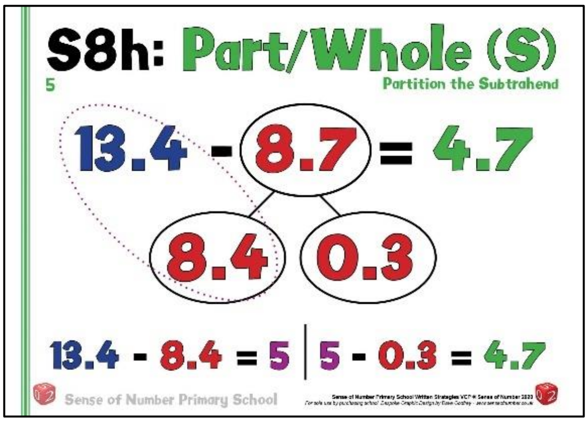


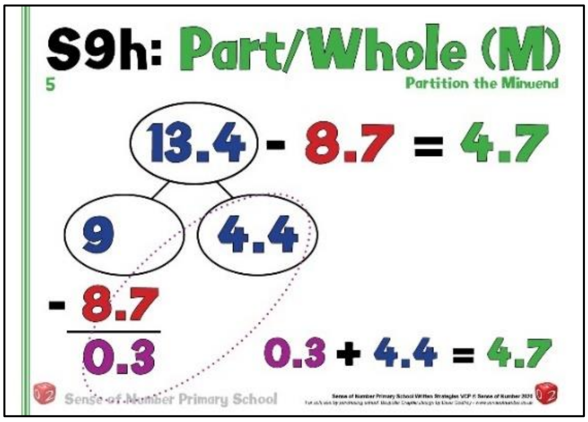


For 3-digit calculations, the same principles apply.
Either the subtrahend or minuend can be partitioned in order for the calculation to be simplified.

The slide above shows that **356** needs to be subtracted From **723**. If the **356** is partitioned into **323** and **33** then the **323** can easily be subtracted from **723**.
This leaves **400**, from which the **33** is then subtracted, giving a final difference of **367**.

If, instead, we partition the minuend of **723** into **400** and **323** then the **356** can be subtracted from 400 (equalling **44**)
The **44** is then added to the **323**, giving an overall difference of **367**





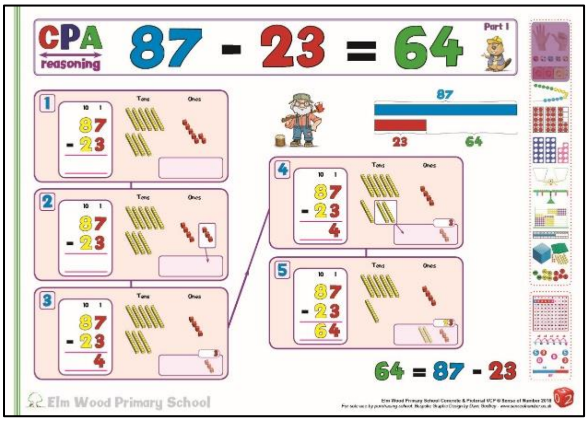
The two approaches even work with decimals. Partitioning the subtrahend of **8.7** into **8.4** and **0.3** means that **8.4** can be easily subtracted from **13.4** (equalling **5**) If we then subtract **0.3** from the **5**, the overall difference is **4.7**

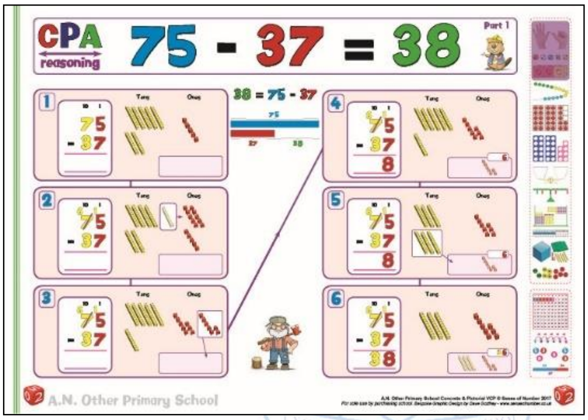
Partitioning the minuend of **13.4** into **9** and **4.4** allows the **8.7** to be subtracted from the **9**. This **0.3** can then be added to the **4.4**, again giving an overall difference of **4.7**

Stage 2	**Expanded Method & Number Lines (continued)**	
	Subtraction by counting back Expanded Method	**Subtraction by counting up Number Lines (continued)**
	In Year 3, according to the New Curriculum, children are expected to be able to use both jottings <u>and</u> written column methods to deal with 3-digit subtractions. **This is only guidance, however – as long as children leave Year 6 able to access all four operations using formal methods, schools can make their own decisions as to when these are introduced.** **It is very important that they have had regular opportunities to use the number line 'counting up' approach first (right hand column below) so that they already have a secure method that is almost their first principle for most 2 and 3-digit subtractions.** This means that once they have been introduced to the column method they have an alternative approach that is often preferable, depending upon the numbers involved. The number line method also gives those children who can't remember or successfully apply the column method an approach that will work with any numbers (even 4-digit numbers and decimals) if needed. It is advisable to spend at least the first term in Year 3 focusing upon the number line / counting up approach as a jotting through regular practice, while resources such as Base 10 are being used to explore decomposition practically. The column method can then be introduced in the 2nd / 3rd term once the understanding is secure. **Ideally, whenever columns are introduced, the expanded method should be practiced in depth (potentially up until 4-digit calculations are introduced). This should be done firstly with apparatus to build up a visual picture, and then gradually developed into the column procedure.**	
Y2/3	**The expanded method of subtraction is an excellent way to introduce the column approach as it maintains the place value and is much easier to model practically with place value equipment such as Base 10 or place value counters.** Introduce the expanded method with 2-digit numbers, but only to explain the process. Column methods are very rarely needed for 2-digit calculations. **Give the children ample opportunity to extend their place value skills into column subtraction**	

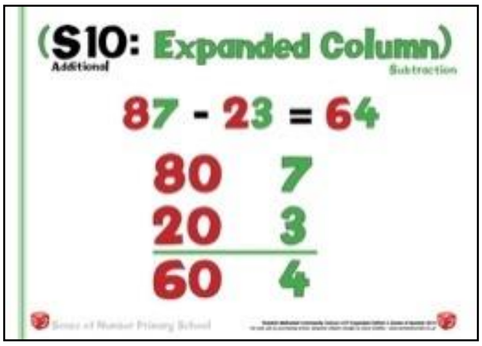
by 'making' the calculation and explaining the process <u>before</u> writing it down.

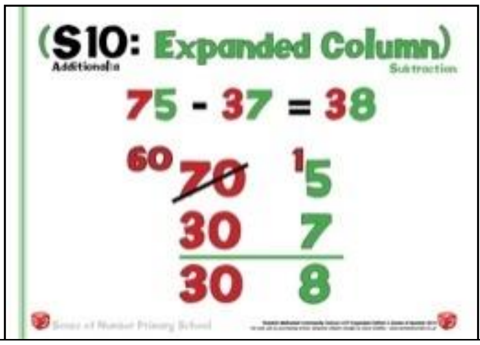
Partition both numbers into tens & ones, firstly with no exchange then exchanging from tens to ones.

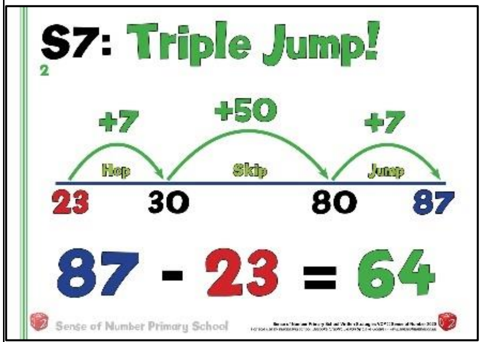


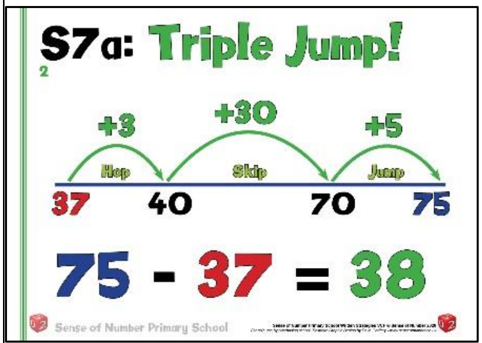


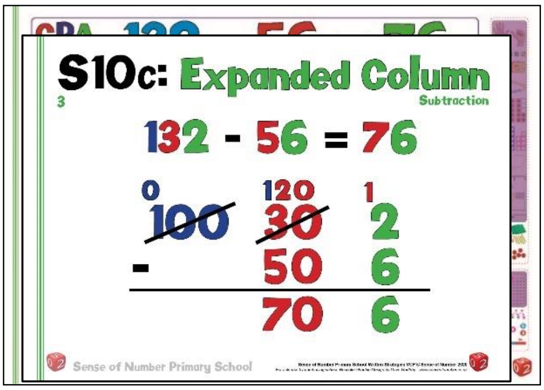
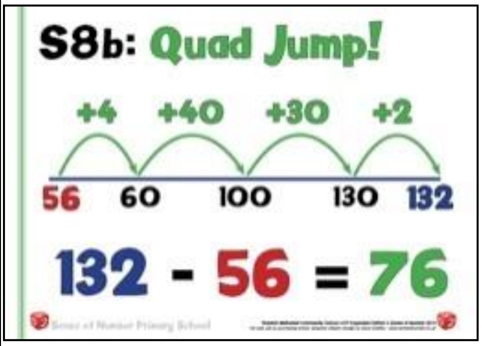
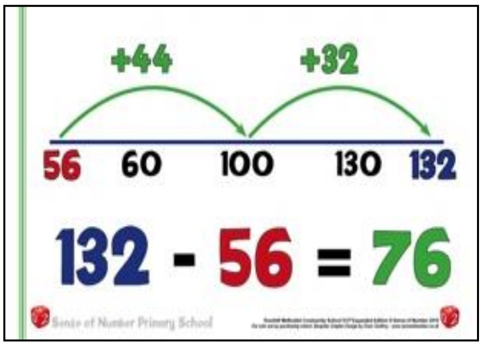
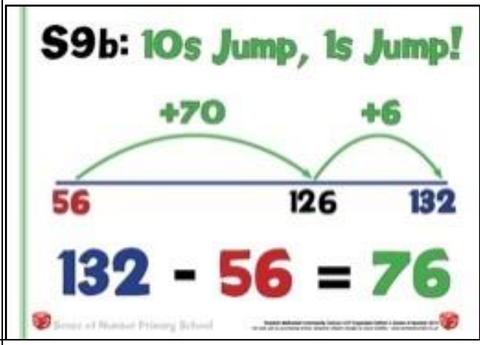
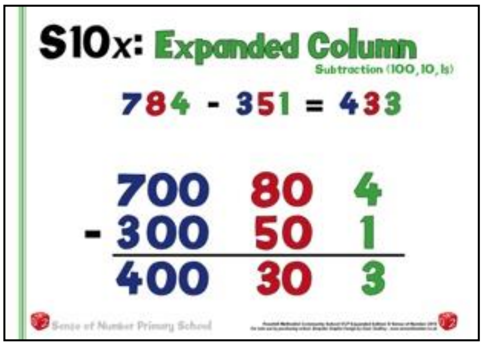
Make sure that children are explaining the process of exchanging / regrouping whilst using the materials. Give them many opportunities to take the 1 Ten and exchange / regroup as 10 Ones before writing this down in expanded form.



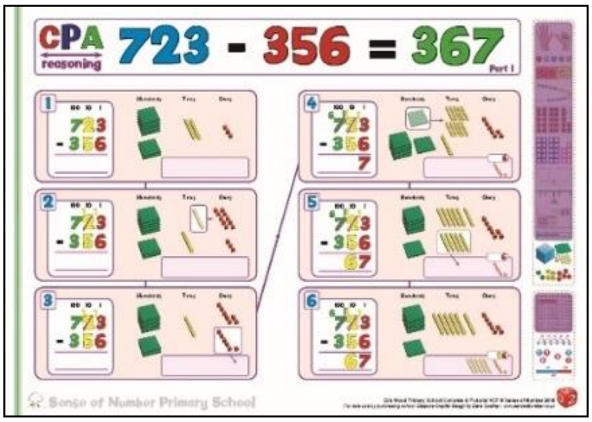


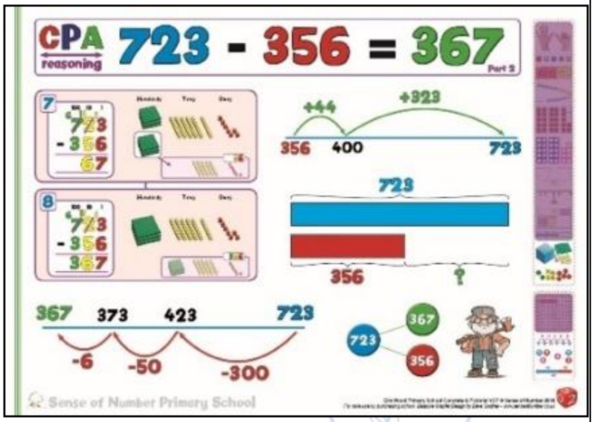


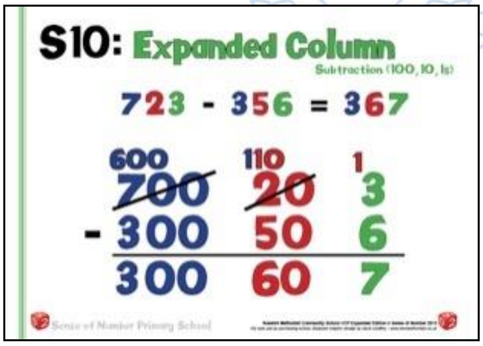


	Develop into exchanging from hundreds to tens and tens to ones, always ‘making’ and discussing with apparatus first. Be explicit in explaining how the apparatus leads to the column procedure. N.B. Please note the layout of Base 10 in Tens Frame style, so that the groups of 5 can be seen immediately and don’t need to be re-counted.  	The number line method is equally as effective when crossing the hundreds boundary, either by the triple / quad jump strategy or by counting in tens then ones.  The ‘quad jump’ can be completed by many children in fewer steps, using either a triple or **double** jump.  
A	Develop the method into three-digit numbers. Subtract the ones, then the tens, then the hundreds.  Demonstrate without exchanging first.	Continue to spot small differences when calculating with 3-digit numbers **E.g. 403 – 397 = 6**

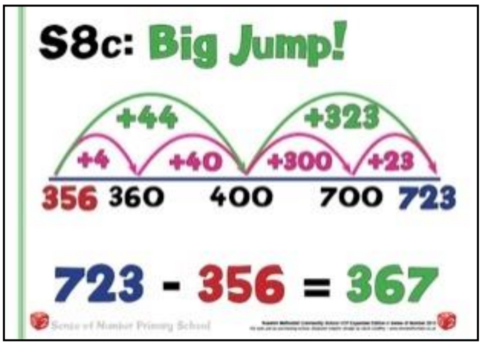
<table>
<tr>
<td>B</td>
<td>Move towards exchanging from hundreds to tens and tens to ones, in two stages if necessary. Use practical apparatus first at all times, especially when dealing with 3-digit calculations.







</td>
<td>The example below shows 2 alternatives, for children who need different levels of support from the image.



</td>
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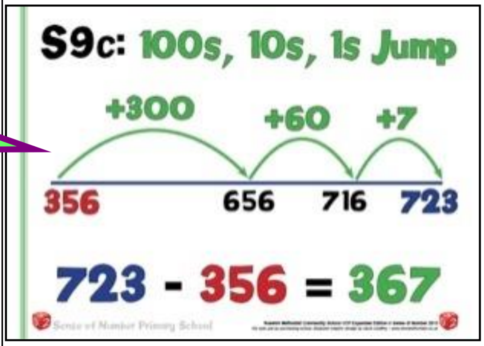
<tr>
<td></td>
<td>For examples where exchanging is needed, then the number line method is equally as efficient, and is often easier to complete</td>
<td>As before, many children prefer to count in hundreds, then tens, then ones.



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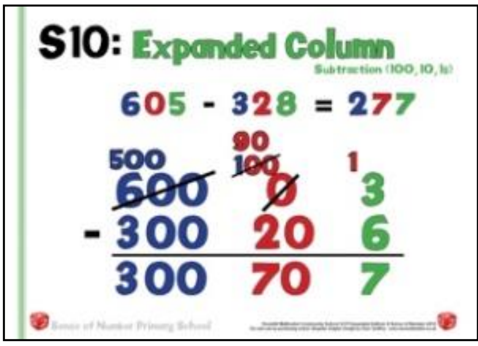
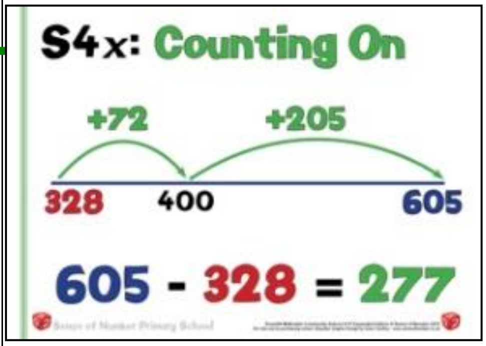
<table>
<tr>
<td>C</td>
<td>Use some examples which include the use of zeros


Continue to use expanded subtraction until both number facts and place value are considered to be very secure!</td>
<td>For numbers containing zeros, counting up is often the most reliable method.

</td>
</tr>
</table>

<table>
<tr>
<th rowspan="2">Stage 3</th>
<th colspan="2">Standard Column Method (decomposition)</th>
</tr>
<tr>
<th>Subtraction by counting back
Standard Method</th>
<th>Subtraction by counting up
Number Lines (continued)</th>
</tr>
<tr>
<td></td>
<td colspan="2"></td>
</tr>
<tr>
<td>Mainly
Y3/4</td>
<td>Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.</td>
<td></td>
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</table>

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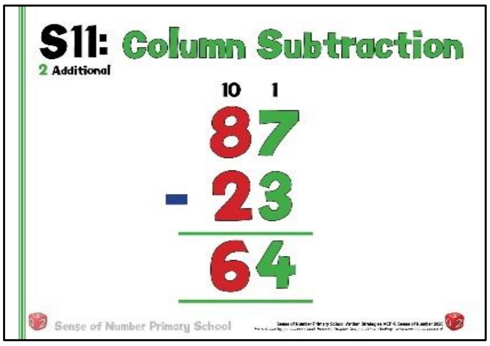
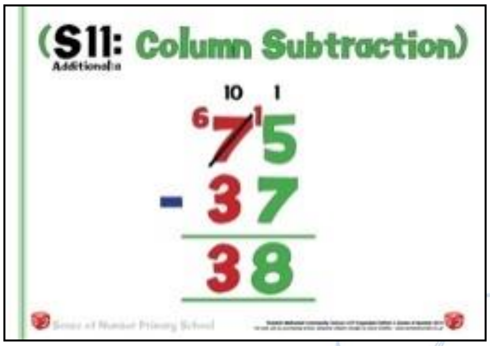
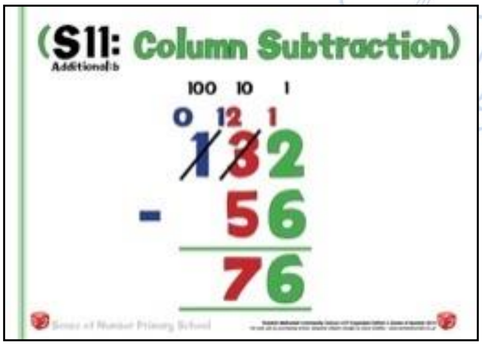
<tr>
<td></td>
<td>

As with expanded method, and using practical resources such as place value counters to support the teaching, children in Years 3 or 4 (depending when the school introduces the column procedure) will quickly move from decomposition via 2-digit number ‘starter’ examples to 2 / 3 digit and then 3-digit columns.







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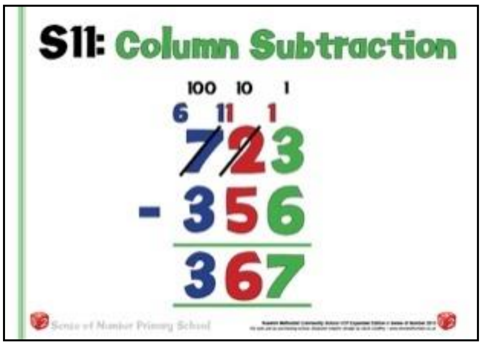
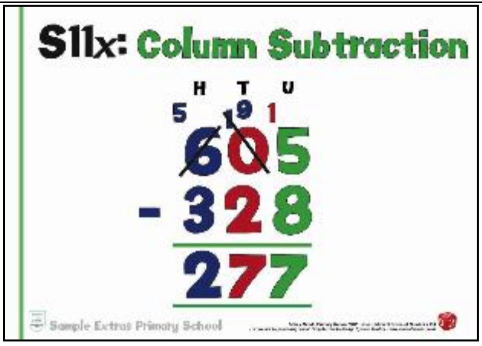
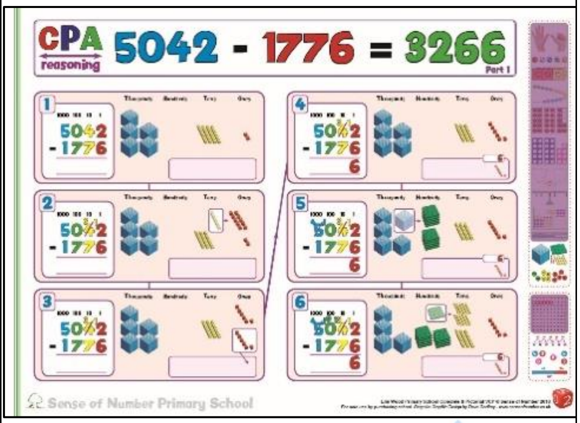
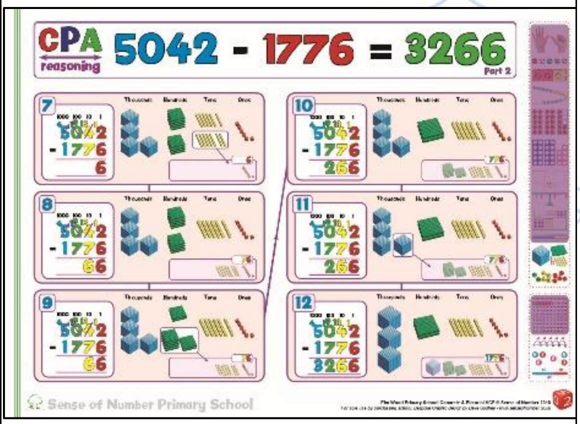
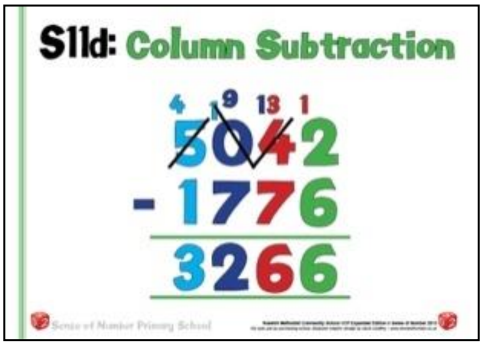
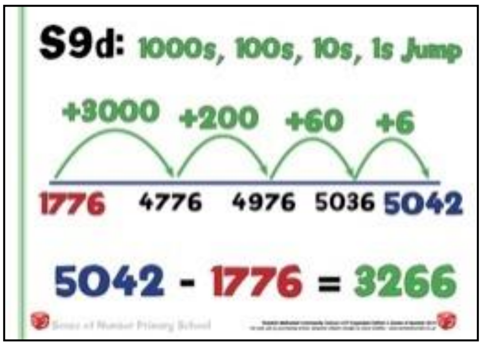
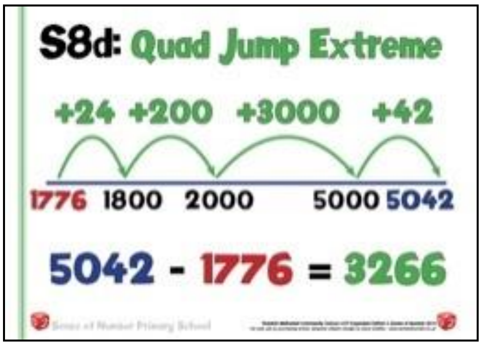
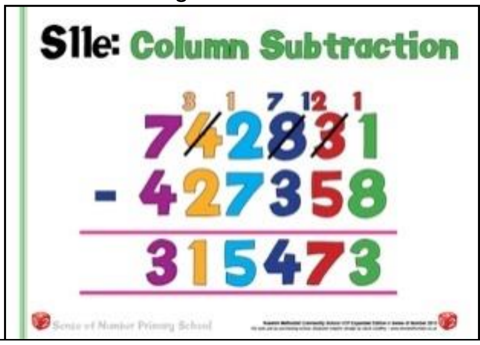
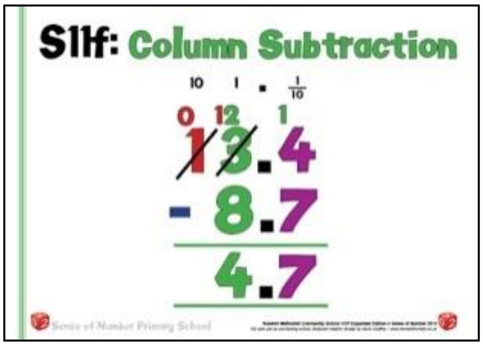


</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2)</td>
<td></td>
</tr>
</table>

		
Y4	From Y4 onwards, move onto examples using 4-digit (or larger) numbers and then onto decimal calculations.   If necessary, apparatus can still be used to demonstrate the exchange / regroup principle. 	 It is even possible, for children who find column method very difficult to remember, or who regularly make the same mistakes, to use the number line method for 4 digit numbers, using either of the approaches 

<table>
<tr>
<td>Y5/6</td>
<td>In Years 5 & 6 apply to examples which use 6 or 7 digit numbers

</td>
<td></td>
</tr>
<tr>
<td colspan="3">Both methods can be used with decimals, although the counting up method becomes less efficient and reliable when calculating with more than two decimal places.</td>
</tr>
<tr>
<td></td>
<td>
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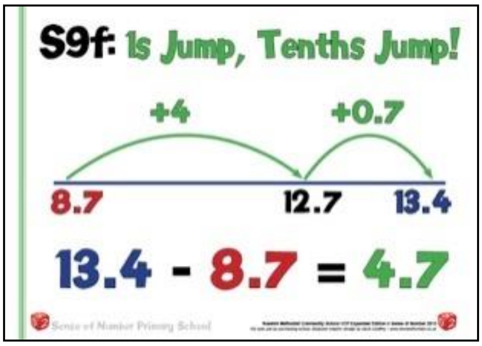
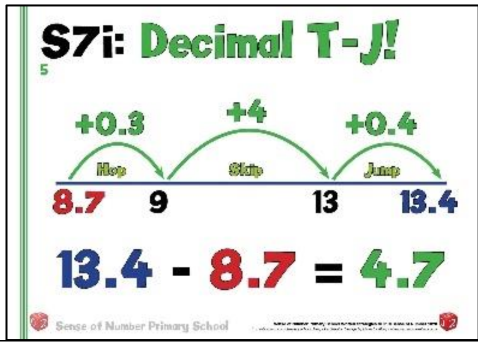
<td>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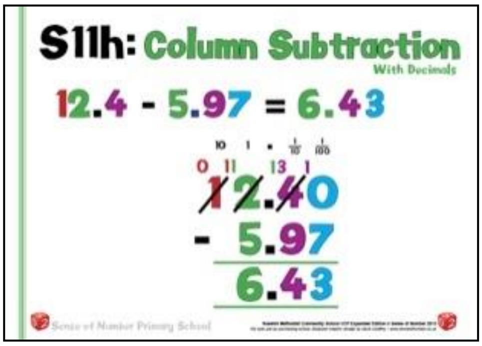
<td>
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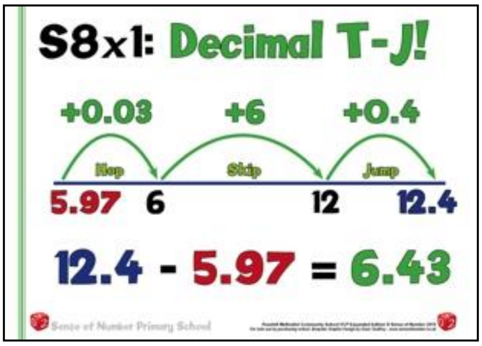
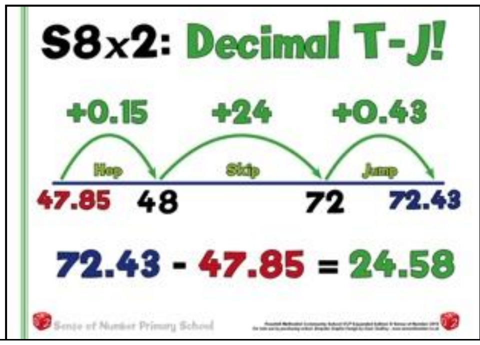
<td>


</td>
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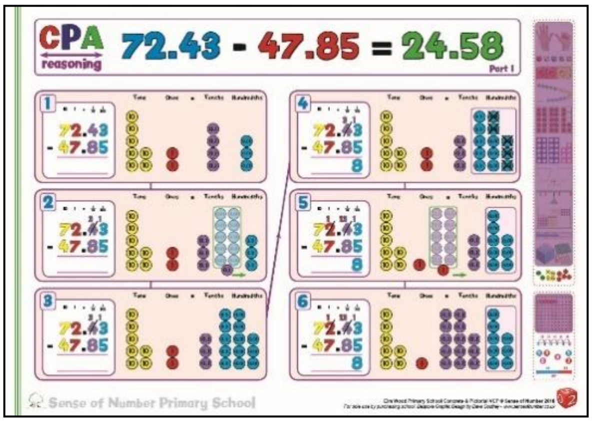
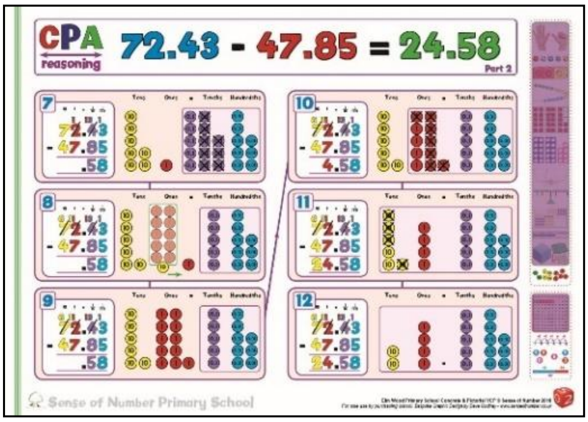
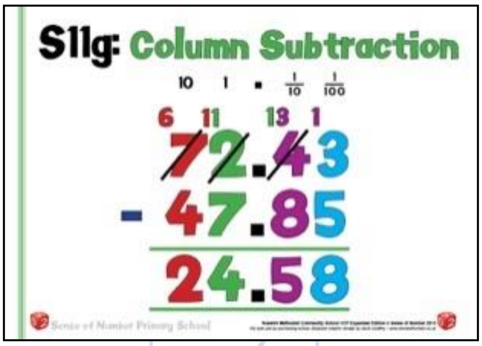
</table>

with calculations to 2 decimal places, practical apparatus can be used initially to explore / embed understanding or at a later stage for the children to demonstrate greater depth. In these instances, they can recreate a column method with decimals and explain each stage of the procedure.

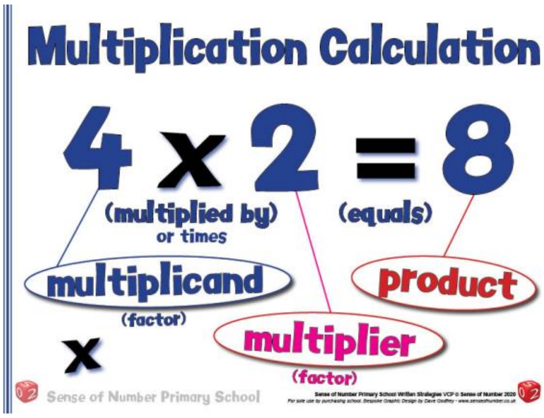






Multiplication Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.



These notes show the stages in building up to using an efficient method for

- 2-digit by 1-digit multiplication by the end of Year 3,
- 3-digit by 1-digit multiplication by the end of Year 4,
- 4-digit by 1-digit multiplication **and** 2/3-digit by 2-digit multiplication by the end of Year 5
- 3/4-digit by 2-digit multiplication **and** multiplying 1-digit numbers with up to 2 decimal places by whole numbers by the end of Year 6.

To multiply successfully, children need to be able to:

- **recall all multiplication facts to 12 × 12;**
- **partition numbers into multiples of one hundred, ten and one;**
- **work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;**
- **similarly apply their knowledge to simple decimal multiplications such as 0.7 x 5, 0.7 x 0.5, 7 x 0.05, 0.7 x 50 using the related fact 7 × 5 and their knowledge of place value;**
- **add two or more single-digit numbers mentally;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **add combinations of whole numbers using the column method (see above).**

Y2

Note:

Children need to acquire **one efficient written method of calculation for multiplication, which** they know they can rely on **when mental methods are not appropriate.**

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

These mental methods are often more efficient than written methods when multiplying.

Y3

Use partitioning and grid methods until number facts and place value are secure

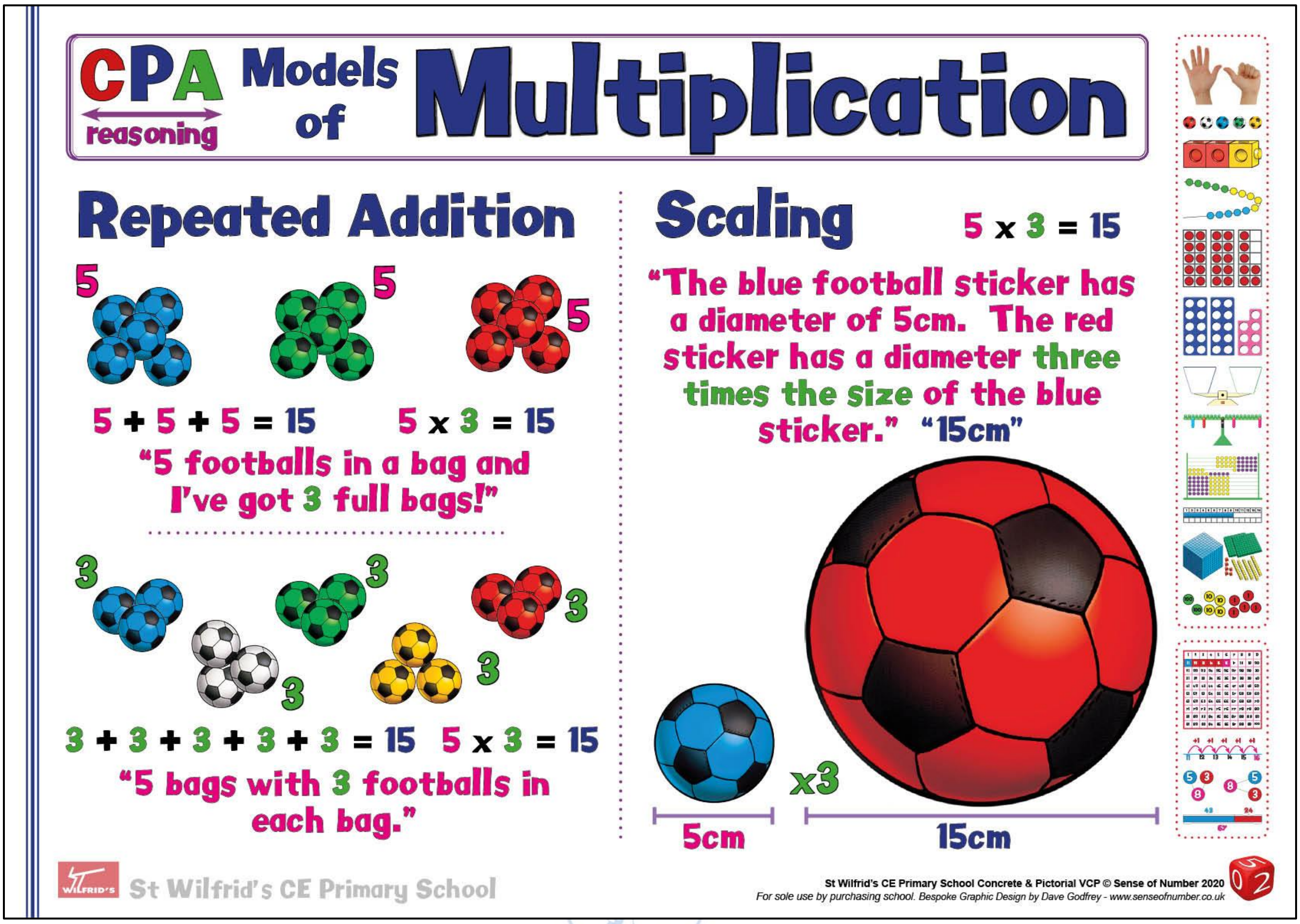
For a calculation such as 25 x 24, a quicker method would be 'there are four 25s in 100 so 25 x 24 = 100 x 6 = 600

When multiplying a 3 / 4 digit x 2-digit number the standard method is usually the most efficient

Y4

At all stages, use known facts to find other facts. E.g. Find 7 x 8 by using 5 x 8 (40) and 2 x 8 (16)

Models of Multiplication



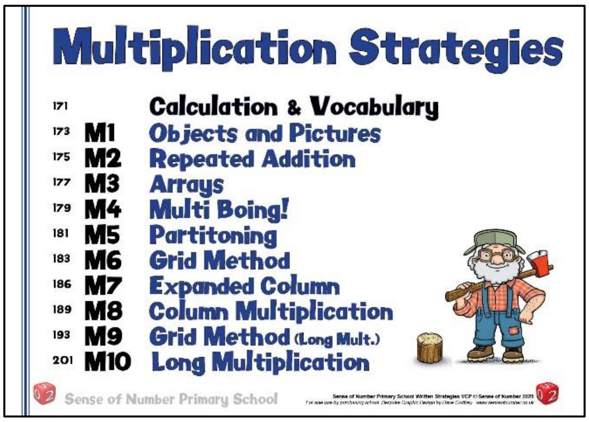
The poster above shows the two images for multiplication which are used within the Visual Calculation Policy. As pictured, the **repeated addition** model can be visualised and explained in two ways (either '**Multiply by…**' or '**Groups of'**)

The other model (**Scaling**) allows the children to see that multiplication is also concerned with making an amount so many times its original size (either larger or smaller)

The reasons for adopting these models are explained below: -

There are **10 key strategies for jottings / written multiplication**, which support the children's understanding, and which can be developed across the year groups.

These can be seen on the right hand poster, and are explained in much more detail, with specific examples outlined, in the remainder of this section on **written multiplication**.



Understanding Multiplication

Children are often asked to draw or make calculations in order to give them a real life context, such as the cupcakes example on the right.

One of the most difficult aspects when teaching multiplication is to ensure that it is taught consistently across the school. Due to the way that most teachers were themselves taught multiplication, there tend to be two approaches adopted across the UK.

Cakes On Plates

Represent 6 x 2

Draw the plates, place the cakes

Finding consistent language & understanding

For a calculation such as 6 x 2, it is commonly viewed either as ‘6 lots of 2’ (see Option A below) or ‘6 repeated three times’ (see Option B).

6 x 2

Teachers in KS1 sometimes move between the two images without realising, but usually settle on one particular explanation. In other schools, there can often be a distinct variation between the model or image adopted in each year group.

In later years, too, the image / explanation taught in KS1 often conflicts with the understanding needed to visualise a higher level calculation in KS2. Therefore, it is important that the images and explanations introduced in KS1 continue to be used and developed in KS2.

Repeated addition - ‘Lots of’ or ‘Repeat a number of times’???

Internationally, when explaining repeated addition, some countries primarily adopt the ‘Groups of…’ image, whilst others teach only the ‘Repeat a number of times / Multiply by…’ model. We are suggesting that actually **both** are models of **6 x 3,** depending on your own interpretation, and it makes sense to introduce both to children as alternate ways of visualising the same calculation.

In effect ‘**1 calculation – 2 interpretations**’ (6 x 2 is ‘**6 groups of 2’** or ‘**6 repeated twice/multiplied by 2)**

Or… **‘1 image – 2 calculations’ (see below)**

The picture on the left could be: -

6 x 2 (6 repeated 2 times / 6 multiplied by 2**)**

Or

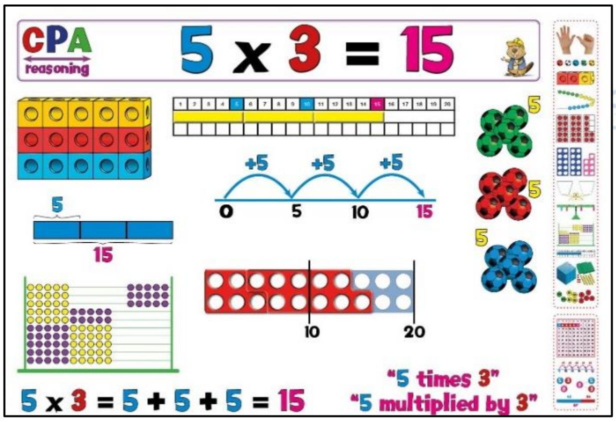
2 x 6 (2 groups of 6)

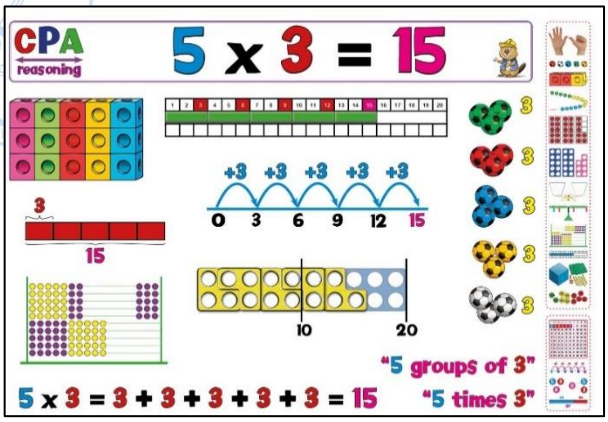
This approach would also correlate to the **Models of Division (Sharing or Grouping)**, which also use two different images to represent the same calculation.

Therefore, in a similar way to aggregation and augmentation for addition having the inverse images of partitioning and reduction for subtraction, both 'multiply by…' and 'groups of…' for multiplication will have an inverse sharing or grouping image for division.

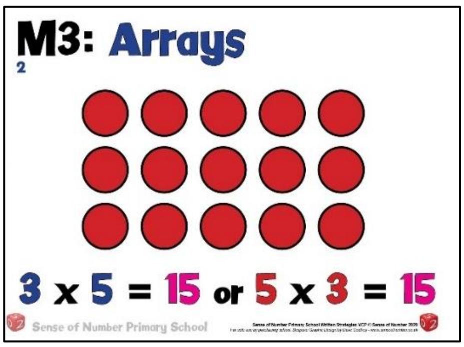
Our policy suggests that initially the most sensible approach, when multiplication is introduced in Year 2, is to ask the children to make / create / display the calculation both ways, so that not only are they experiencing 2 ways to represent a calculation, they are also realising that **multiplication is commutative.** Whichever way they represent **6 x 2 or 5 x 3 (or 2 x 6 or 3 x 5),** they will always get the same product.

Children should use a range of manipulatives, both real life (such as the footballs) or generic materials (such as cubes or number shapes). Each piece of apparatus or pictorial image (e.g. abacus or number line) can be used to show both elements of multiplication.





This reasoning would then lead perfectly into the **array** image, where the calculation can be displayed to show commutativity, and can be read using either interpretation.



The array shows 3 x 5 because it has a column of 3 multiplied by 5 / repeated 5 times.

It also shows 3 groups of / rows of 5.

The array shows 5 x 3 because it has a row of 5 multiplied by 3 / repeated 3 times / trebled.

It also shows 5 groups of / rows of 3

Multiplication In Key Stage 2 – Should ‘Groups Of…’ or ‘Repeat A Number Of Times…’ Be The Default Model?

As mentioned, although both images of a multiplication calculation are valid, and should both be used in KS1, the question still remains as to which should be the ‘default’ method as the children progress through KS2 and are faced with more complex calculations.

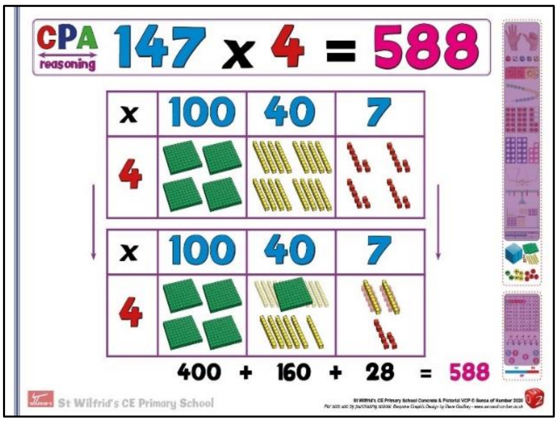
We feel that to answer this, considering we are dealing with multiplication, the key question to ask is: - ‘*Which image is actually showing 5* ***multiplied by*** *3?’*

Using the 2 posters above, the first poster clearly displays ‘**multiply by’** (ie it shows how **5 x 3** means ‘*take 5 objects then* ***multiply*** *the 5 objects by 3’*, showing us ‘**5 three times’** ‘**5 multiplied by 3**, or **5 trebled** Mathematically, therefore, it is a more accurate representation.

If we use the correct vocabulary and regularly say ‘**multiplied by’** (rather than ‘**times’**) when reading a multiplication calculation, then the first set of images would be the ones which would come to mind.

For example, ‘**10 multiplied by 4’** would mean **10 objects (**fingers on hands or beads on an abacus**) repeated 4 times**.

Of equal importance are the types of calculation that children are asked to deal with in KS2. The accepted, conventional way in which a calculation is written is to multiply a 2 / 3 / 4 digit number by a single digit number, not the other way round. E.g. You would usually write 147 x 4, rather than 4 x 147.



Again, this would lead to ‘multiply by’ as the preferred method or way of thinking.

To explain the calculation on the left, you would say ‘*147 multiplied by 4’* and would then display 1 hundred, 4 Tens and 7 Ones, 4 times each.

It would be very inefficient to read the calculation as ‘147 groups of 4’, which would require far too much apparatus and would take an extremely long time to complete.

Hence, our advised approach is to begin with both images and interpretations for a given calculation, develop understanding of commutativity, but to focus predominantly on ‘multiply by’ as the default interpretation for the majority of calculations in KS2.

Scaling

The other way in which multiplication can be explained correctly is through the use of **scaling**.

In effect, 5 x 3 means 5 **scaled up** to be '**3 times the size'.**

This interpretation is commonly used throughout Europe and means that children see a multiplication calculation and immediately picture the answer as something which has been scaled up or down in size.

This can be used with amounts (*I have 5 cakes but my brother Dave has 3 times as many as me)***,** where we actually end up with 20 cakes (not 15) on display.

Me

Dave

Scaling is more commonly used for measurement, though, where the answer to the word problem is '15' but there aren't actually 15 objects.

For example, **5 x 3** could be used for a word problem such as '*John has an 5cm piece of string. Louise has a piece of string which is 3 times as long. How long is Louise's string?'*

In this instance there is no repeated addition **or** 'lots of'. Louise's string isn't 5cm repeated 3 times and it certainly isn't 5 lots of 3 cm (although both would give the same answer if they were laid out end to end).

It is a single 15cm piece of string which is 3 times longer than John's 5cm piece.

Scaling is also very helpful for multiplying by fractions (either way round)

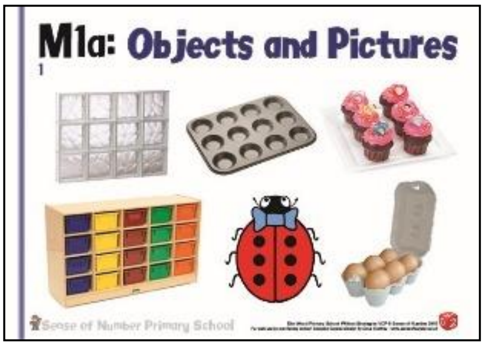
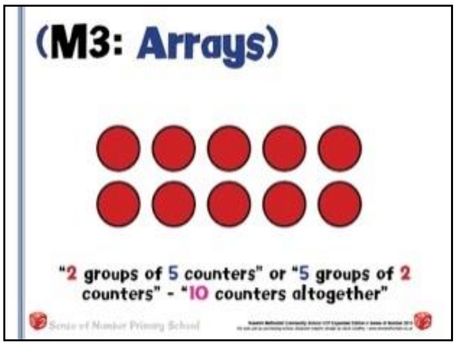
E.g. 12 x 1/3 would mean 12 scaled to be 1/3 of the size (i.e. 3 times smaller)

1/3 x 12 would be 1/3 made 12 times as big or 1/3 repeated 12 times.

Both methods would give the correct answer of 4.

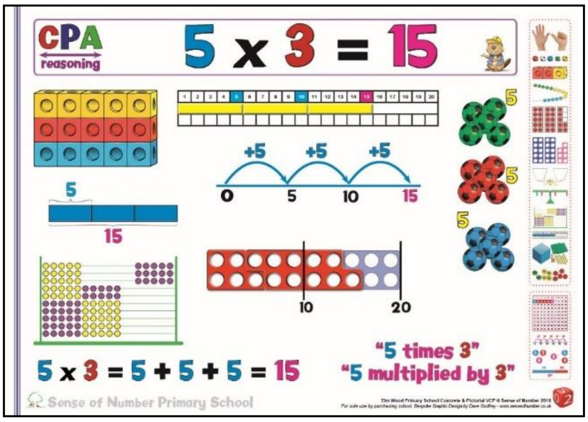
To summarise, here are the various definitions associated with multiplication.

.Written Methods of Multiplication

Stage 1	Number Lines, Arrays & Mental Methods
EYFS	In Early Years, children are introduced to grouping, and are given regular opportunities to put natural resources and real life objects into groups of any given size They also stand in different sized groups, and use the term ‘pairs’ to represent groups of 2. They are expected to ‘subitise’ small amounts so that they can immediately recognize randomly arranged groups of 2, 3, 4 and 5 Later in the year this is developed into using more structured resources such as Ten Frames, Number Shapes, multi-link cubes or an abacus. Children use real life resources (then 5 / 10 Frames or Number Shapes) to begin to count in ones, twos, fives and tens, saying the multiples as they count the pieces. E.g. Saying ‘2, 4, 6, 8, 10 etc’ or ‘1 pair, 2 pairs, 3 pairs etc) when counting pairs of shoes Also ’10, 20, 30’ or ‘1 Ten, 2 tens, 3 tens’ whilst counting Tens Frames / Fingers
Y1	 Begin by introducing the concept of multiplication as repeated addition. Before using mathematical apparatus, use real objects and equipment such as cups, cakes, footballs, pencils, apples etc.) Children will firstly make then draw these objects in groups giving the product by counting up in 2s, 5s, 10s and beyond, and finally by writing the multiplication statement. The picture above will begin as an addition ‘story’ (**5 footballs and 5 footballs make 10 footballs**) but will then be written as a multiplication calculation (**5x 2 = 10**) or **(2 x 5 = 10)** **Make sure from the start (**as explained in the introduction to this section) **that all children say the multiplication fact both ways way round, using the word ‘multiply by’ to interpret the above picture as 5 x 2 or ‘groups of’ to interpret it as 2 x 5.** Ie. There are **5 footballs in 2 groups**, showing **5 multiplied by 2 (5x2),** as well as **2 groups of 5 footballs (2 x 5)**
Y2	**The array**  Build on children’s understanding that multiplication is repeated addition, using both interpretations (‘Multiply by’ and ‘Groups of’) To do this, begin to use arrays and number lines to support their thinking. Start to develop the use of the array to show linked facts (commutativity). Make arrays with a wide range of objects, especially those which naturally occur in real-life such as windows, egg boxes, drawers or cake trays. Emphasise that all multiplications can be worked out either way (2 x 5 = 5 x 2 = 10) as this will support the children in the future learning of their tables facts.

Practice counting in both steps (E.g. Both 2s and 5s) to prove that the answer is the same no matter which way round they are counted.

Encourage the children to see how the array makes it easier to see the calculation as a multiplication rather than a repeated addition.



As with addition & subtraction, the key to developing clear understanding of multiplication is to involve the children in visualizing calculations with a wide range of resources.

Using manipulatives such as Numicon, abacuses, number rods & multi-link, children can demonstrate & discuss their interpretation of multiplication statements

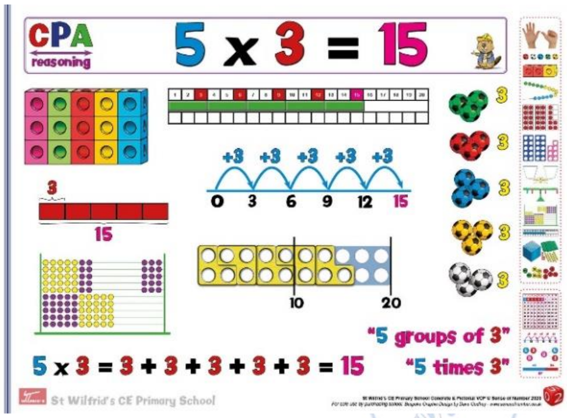
Continue to emphasise multiplication both ways round.

E.g. 5 x 3 = 5 + 5 + 5

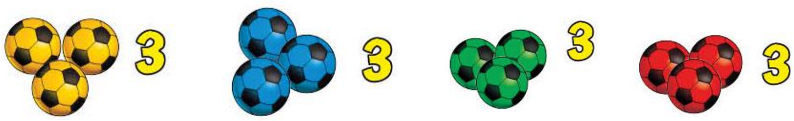
5 multiplied by 3 = 15 or

5 x 3 = 3 + 3 + 3 + 3 + 3

5 groups of 3 = 15



Again, using resources and real life objects, begin to involve the children in telling stories and creating pictures for the 3s and 4s, (as well as 2's, 5's and 10's), and then into writing multiplication statements using either interpretation.

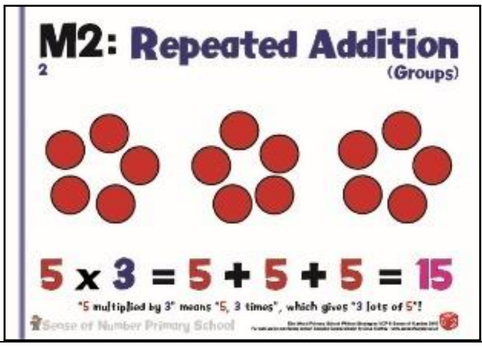


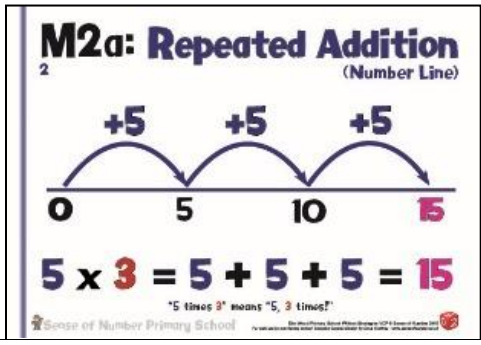
E.g. **3 footballs + 3 footballs + 3 football + 3 footballs would show 12 footballs**

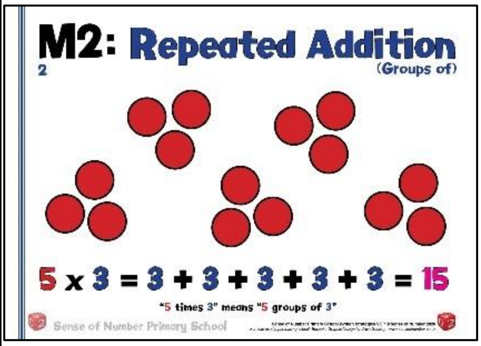
3 footballs multiplied by 4 / 3 footballs 4 times = 12 footballs (3 x 4 = 12)

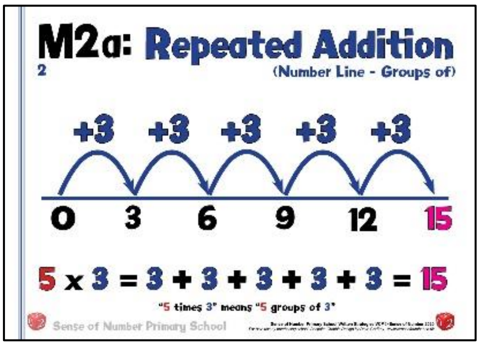
4 groups of 3 footballs = 12 footballs (4 x 3 = 12)

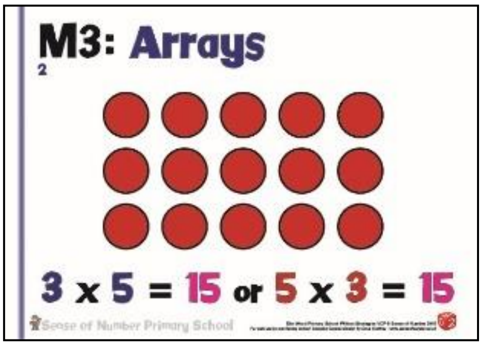
Manipulatives can then be used alongside simple pictures and jottings, such as circles, number lines and arrays, which support the transition from repeated addition to multiplication.





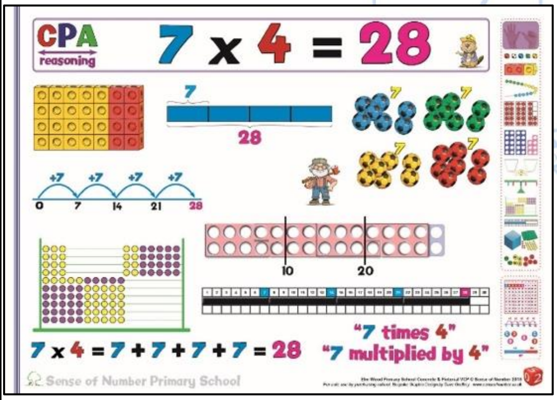


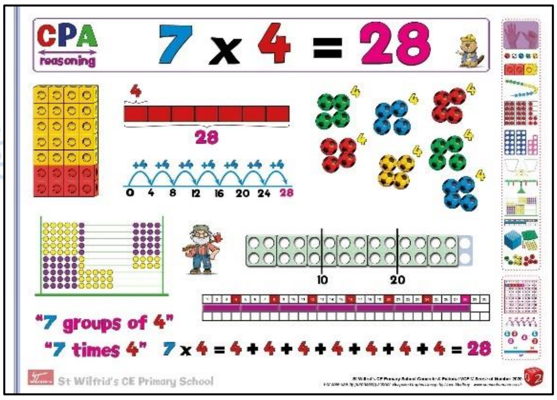


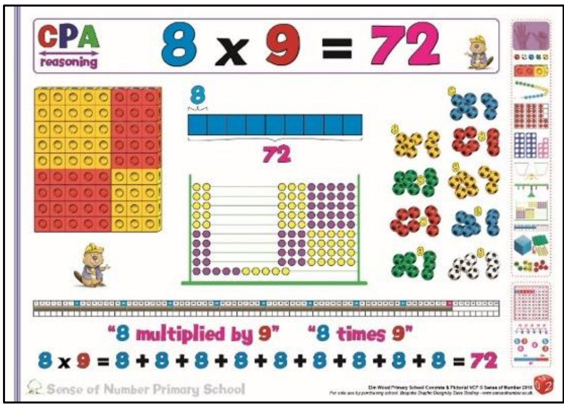


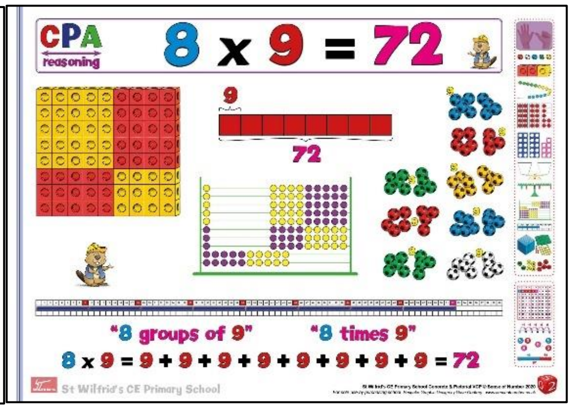
Y3 & Y4

Extend the above to include the **3, 4, 8 and 11** times tables (**Year 3**)
then the **6, 7, 9 & 12 times** tables (**Year 4**)









Even when dealing with larger tables such as the 7's and the 8's it is crucial that children can create a model or image so that they can understand what the calculation actually means

The footballs simply show 4 Sevens / 7 Fours and 9 Eights / 8 Nines as a real life picture, but the other images support children in actually either seeing the final product or understanding the calculation (and how the product is visualised) in more depth.

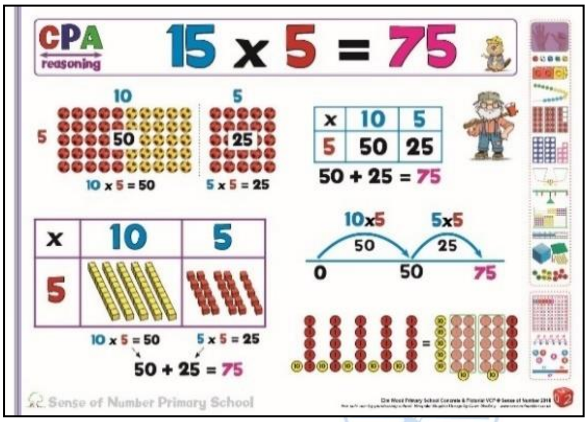
The **Multi-link** and **Abacus** images are especially useful as they allow the **7 times table** to be seen as a combination of the **5s and 2s** and the **8 times table** as **5s and 3s.**

E.g. the 7 x 4 multi-link / abacus images are colour-coded to show a 5x4 array and a 2x4 array. The 8 x 9 images show 3x9 and 5x9.

The Number Rod image places the 4s / 7s / 8s /9s on a track to display the products 28 / 72, whilst the Numicon pieces are arranged on a Numicon 'track' to show the same product.

If the children become accustomed to using resources to support their counting, then use similar resources when displaying a visual for multiplication, they will automatically feel more confident when asked to picture / create / visualize a higher level calculation such as those later in this policy.

Extend the use of resources for 2 digit x 1 digit calculations so that children can visualize what the calculation looks like before they are taught any specific written methods or jottings.



As mentioned earlier in the policy, it is at this stage where we only use the '**multiply by…**' interpretation of the calculation for efficiency.

'15 multiplied by 5' allows us to represent the calculation by showing 15 five times.

'15 groups of 5' would achieve the same answer, but would take much longer to visualize and complete

In each of the images above, **15 x 5** can be shown as a basic Tens and Ones partition. (**I.e. 10 multiplied by 5 and 5 multiplied by 5**) but the images allow different visualisations.

The **footballs** are laid out like a **Slavonic abacus**, allowing a clear visual of all 75 footballs but in an arrangement which makes them very easy to calculate.

The **Base 10 apparatus** is probably the most important image, and shows how the Tens and Ones are actually **partitioned** within a Grid Method – **5 Tens and 5 Fives.**

This is the model (along with the Place Value Counters) which the children need to 'make' most often in class so that they become accustomed to exchanging / regrouping **Ones** into **Tens** (and then **Tens** into **Hundreds** with more complex calculations)

The **Place Value Counters** then demonstrates how the Tens and Ones are **regrouped**. **Twenty** of the **Ones** are exchanged / regrouped into **2 Tens**, giving **7 Tens and 5 Ones.**

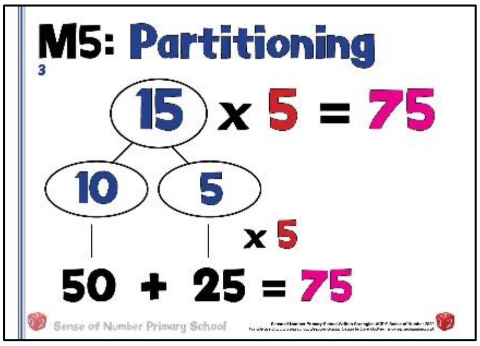
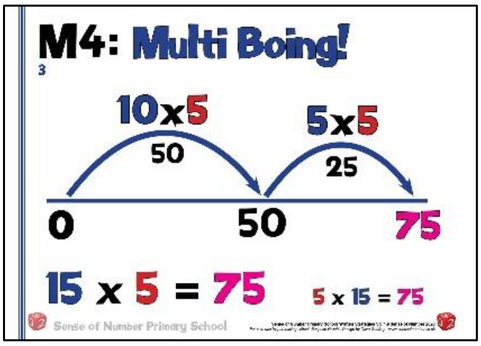
The Grid Method (covered in detail later) and number line (see below) are also pictured on the poster, and are very helpful jottings that allow children to quickly show their strategy.

Y3

Once the general models and images for multiplication are secure,

begin to partition the 2 digit number using **jottings and number lines.**

Extend the methods above to calculations which give products greater than 100.

	  ***Each of these methods can be used in the future if children find expanded or standard methods difficult.***

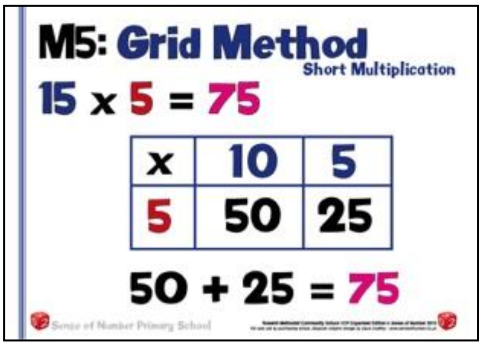
Use of ‘Grid’ Method within the 2014 Curriculum

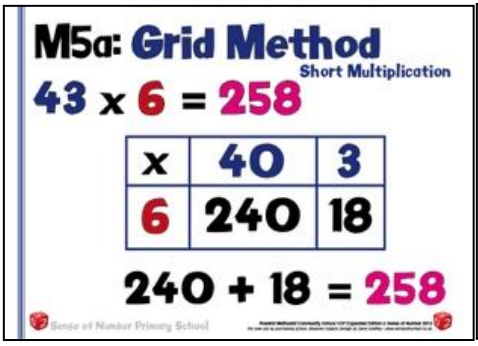
In the 2014 Curriculum, the Grid Method is <u>not</u> exemplified as a written method for multiplication. The only methods specifically mentioned are column procedures.

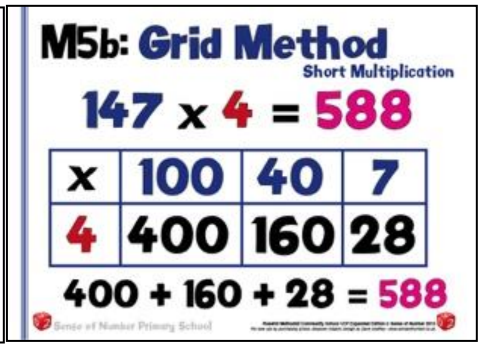
Most schools in the UK, however, have effectively built up the use of the grid method over the past 20 years, to the extent that it is generally accepted as one of the most appropriate ‘written’ methods for simple 2 and 3 digit x single digit calculations (**short multiplication**).

It develops clear understanding of place value through partitioning as well as being an efficient method, and is especially useful in Years 4 and 5.

Consequently, grid method is a key element of this policy, but, to align with the 2014 Curriculum, is classed as a mental ‘jotting’. It builds on partitioning, and is also the key mental multiplication method used by children in KS2 (pg. 44 – Multiplication Partitioning)



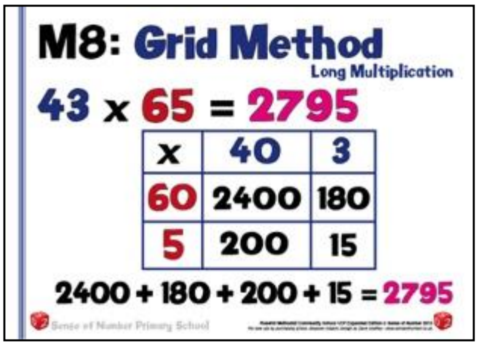




The examples above show the development of grid method from a straightforward 2 digit x 1 digit calculation with a product below 100 (15 x 5) to a more complex version (43 x 6) and then a 3 digit x 1 digit calculation (147 x 4). In each example the partitioning / place value link is very clear.

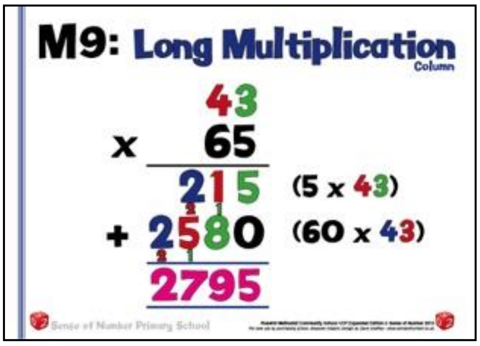
The Grid Method is equally as efficient as column method for almost all 2 digit x 2 digit calculations, providing an excellent basis and security for simple examples of long multiplication.

Many children who find the column procedure difficult when faced with long multiplication are far more successful with the Grid Method, which builds on all of the place value and partitioning work developed earlier.



Column procedures still retain some element of place value, but, particularly for long multiplication, tend to rely on memorising a ‘method’, and can lead to many children making errors with the method (which order to multiply the digits, when to ‘add the zero’, dealing with the ‘carry’ digits’ etc.) rather than the actual calculation.

In these instances, children will continue to use the grid method



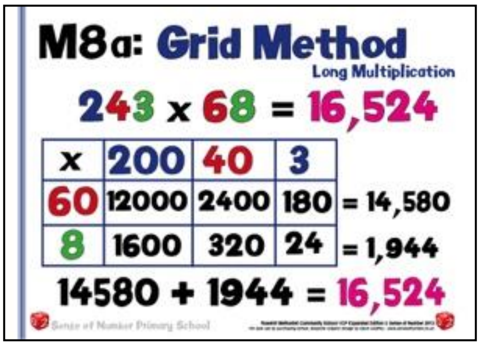
Once the calculations become more unwieldy

(**4 digit x 1 digit** or **3 / 4 digit x 2 digit**

such as the calculation on the right)

then grid method begins to lose its effectiveness,

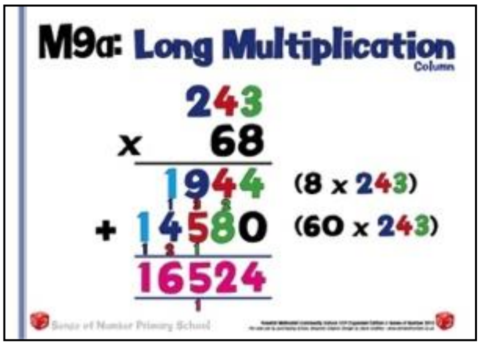
as there are too many zeroes and part products to deal with.



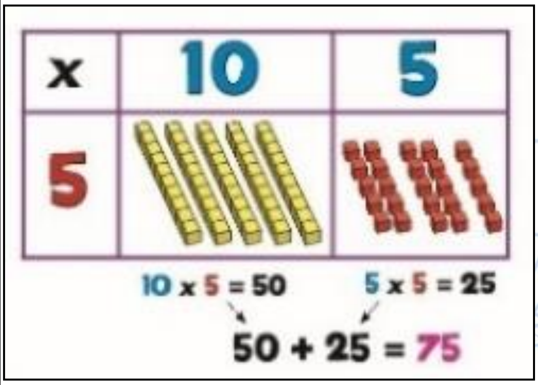
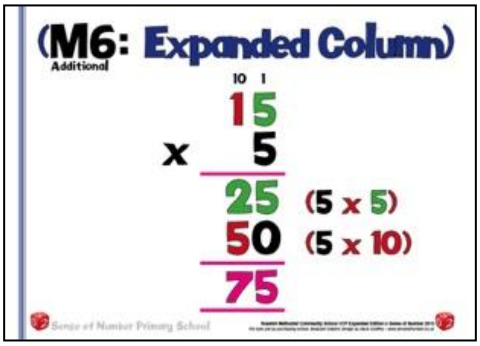
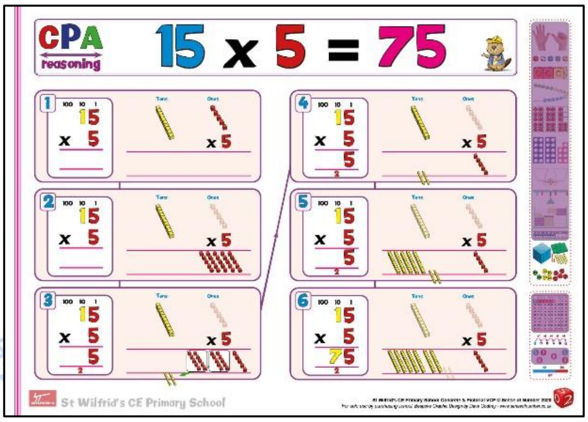
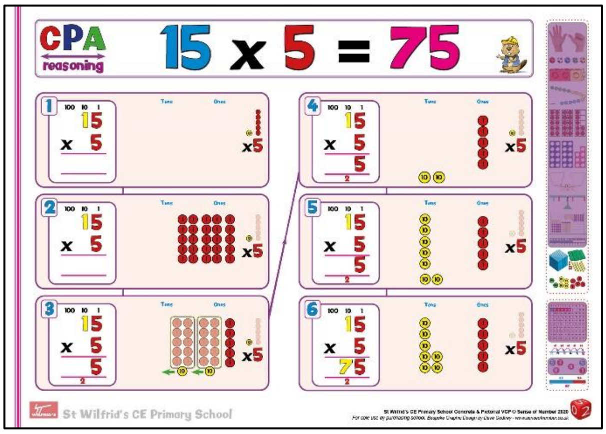
At this stage column procedures are far easier, and,

once learnt, can be applied much quicker.

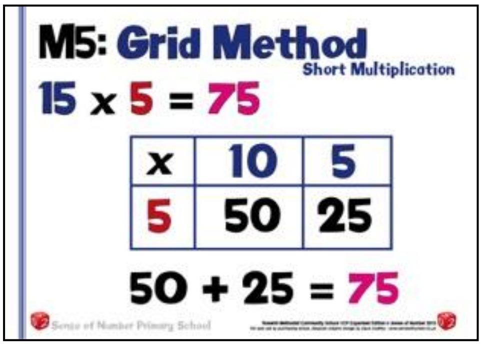
Grid methods can still be used by some pupils who find columns difficult to remember, and who regularly make errors, but children should be encouraged to move towards columns for more complex calculations



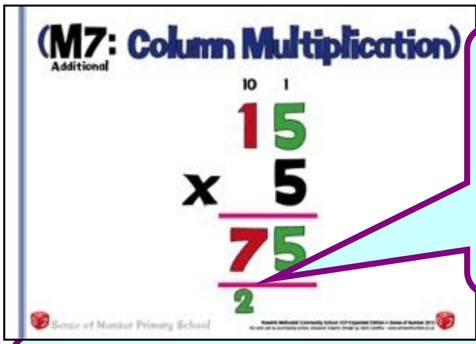
Stage 2	**Written Methods - Short Multiplication**	
	Grid Multiplication (Mental ‘Jotting’)	**Column multiplication** (Expanded method into standard)

Y3	The **grid method** of multiplication is a simple, alternative way of recording the partitioning jottings shown previously. *As shown earlier, it can initially be taught using an array to show the actual product* Begin by using Base 10 to visualize the calculation, allowing the children to see exactly what it actually means.  	The expanded method links the grid method to the standard method. It still relies on partitioning the tens and units, but sets out the products vertically. Children use the expanded method until they can securely use and explain the standard method.  When teaching both expanded and then standard method, always allow the children time to visualise the calculation stages using Base 10   Once Base 10 understanding is secure then begin to use place value counters to represent the same calculations

It is recommended that the grid method is used as the main method within Year 3. It clearly maintains place value, and helps children to visualise and understand the calculation better.

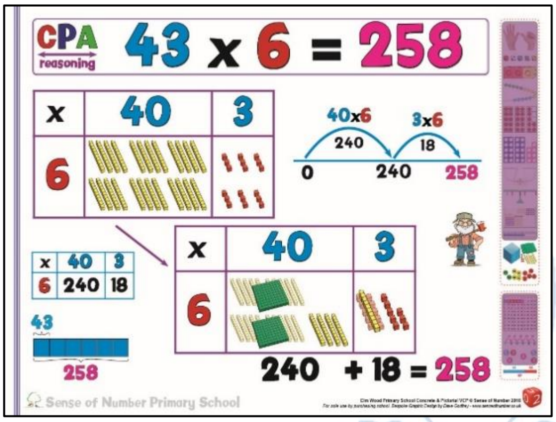


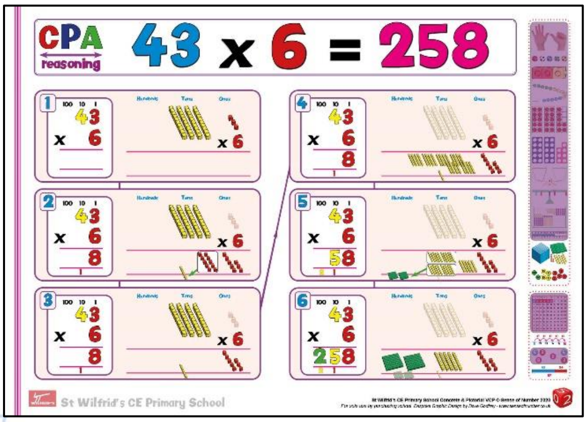
At some point within the year, the column method can be introduced, and children given the choice of using either grid or standard. Some schools may delay the introduction of column method until Year 4



Place the 'carry' digit below the line

When setting out calculations vertically, begin with the ones first (as with addition and subtraction).



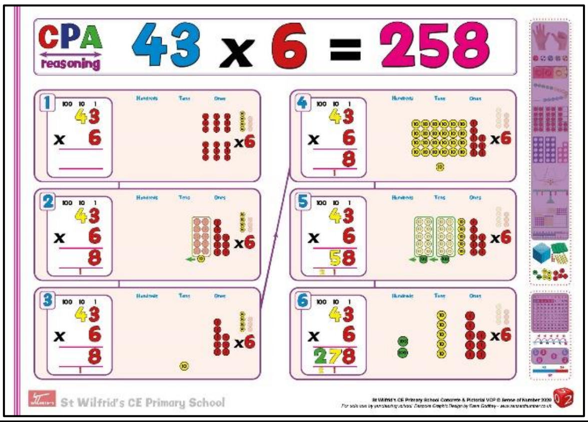


It is important to continue adopting the CPA approach by making and explaining the more complex 2 digit x 1 digit calculations.

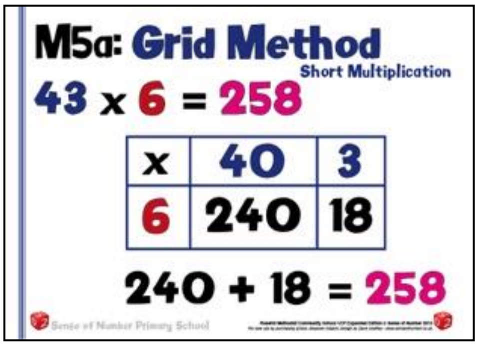
E.g. Using Base 10 apparatus, children can demonstrate **43 x 6** as **4 Tens** repeated **6 times** and **3 Ones** repeated **6 times**. They can actually display this image within a Grid Method and then exchange / regroup **20 Tens** into **2 Hundreds** and **10 Ones** into **1 Ten.**

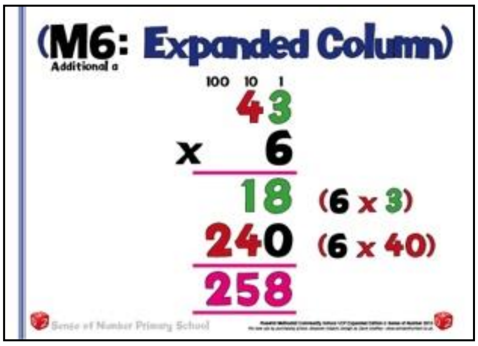
They can also explore and explain the column method using Base 10, this time regrouping the **Ones** first then exchanging for a **Ten** (and putting it in the Tens column) before regrouping the **20 Tens** and exchanging them for **2 Hundreds** (which are put into the **Hundreds** column)

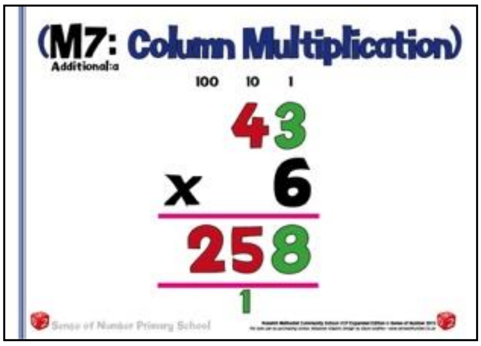
As before, place value counters can then be used to help explain the column method in a slightly more abstract way, but one which still maintains a secure understanding of the underlying concepts.



Y4

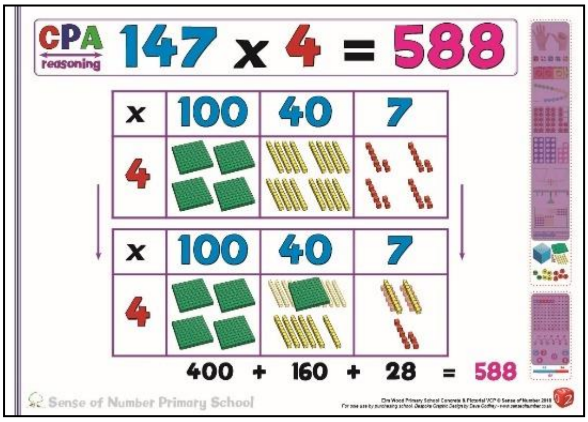






Continue to use **both grid** and **column methods** in Year 4 for more difficult **2 digit x 1 digit** calculations, extending the use of the grid method into mental partitioning for those children who can use the method this way and can simply jot down the sub products and answer.

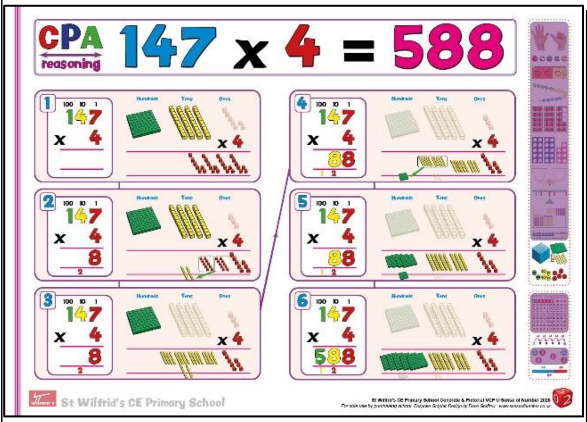
At this point, the expanded method can still be used when necessary (to help 'bridge' grid with column), but children should be encouraged to use their favoured method (grid or column) whenever possible.

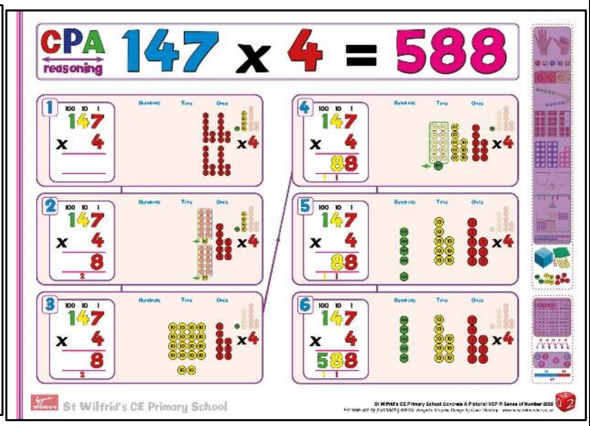


Using **apparatus** for **3 digit x 1 digit** calculations is an excellent way to continue developing the conceptual understanding of what the calculations look like and the actual size of the number that is being manipulated.

In the first example displayed, children can create **100 x 4, 40 x 4 and 7 x 4** using Base 10, then show how the **16 Tens** can be regrouped then exchanged / into **1 Hundred and 6 Tens**, while the 28 Ones can be regrouped then exchanged into 2 Tens and 8 Ones.

In the examples below, Base 10 and Place Value Counters are used to show the step-by-step process involved in a column method, beginning with the 7 Ones being multiplied by 4 then regrouped / exchanged (with the 2 Tens being placed in the Tens column). This is followed by the 4 Tens being multiplied by 4 and regrouped / exchanged (with the 1 Hundred being placed in the Hundreds column). Finally the 1 Hundred is multiplied by 4 and the 5 Hundreds are totalled.





<table>
<tr>
<td></td>
<td colspan="2">For 3 digit x 1 digit calcualtions, both grid and standard methods are efficient.
Continue to use the grid method to aid place value and mental arithmetic.
Use column method for speed, and to make the transition to long multiplication easier.
If both methods are taught consistently then children in Year 4 will have a clear choice of 2 secure methods, and will be able to develop both accuracy & speed in multiplication.</td>
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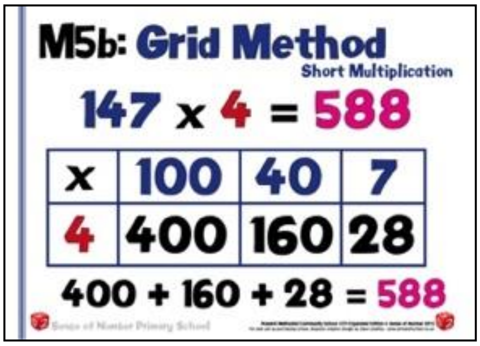
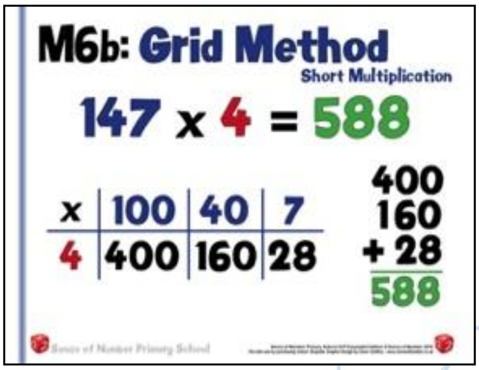
<td></td>
<td>




Sometimes children find the multiplication and place value parts of Grid Method to be fairly simple, but then struggle with the actual addition at the end (see 1st example above).
In these instances, encourage them to complete the addition using a column method (see 2nd example above)
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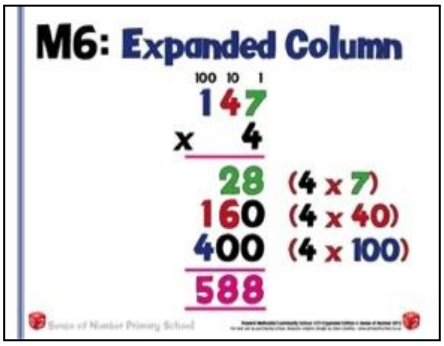
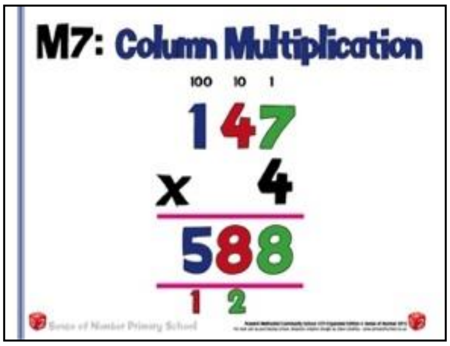
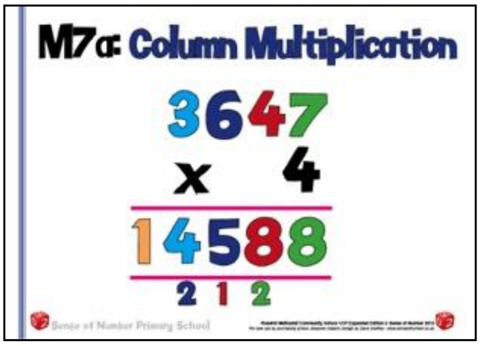


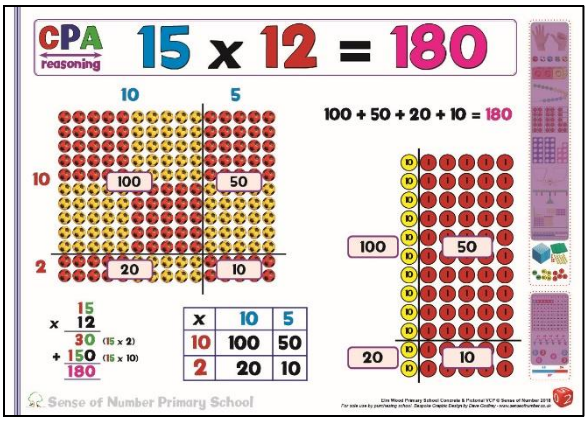


Expanded method can still be used for children who need extra support with place value
(and as a ‘bridging’ method between
grid and column procedures)
Once this is secure then they can practice the speed and security of the column method, but ensuring they can still explain the place value (using apparatus if necessary) when required
</td>
</tr>
<tr>
<td>Y5</td>
<td>For a 4 digit x 1 digit calculation, the column method, once mastered, is quicker and less prone to error.
The grid method may continue to be the main method used by children who find it difficult to remember the column procedure, or children who need the visual link to place value.</td>
<td>


</td>
</tr>
</table>

Stage 3 — Long Multiplication (TU x TU)

Stage 3	Grid Multiplication	Column multiplication (Expanded method into standard)



Even a simple long multiplication calculation can be displayed visually using an array set out in a Slavonic Abacus arrangement. The image on the poster shows a straightforward away to quickly see all 180 counters

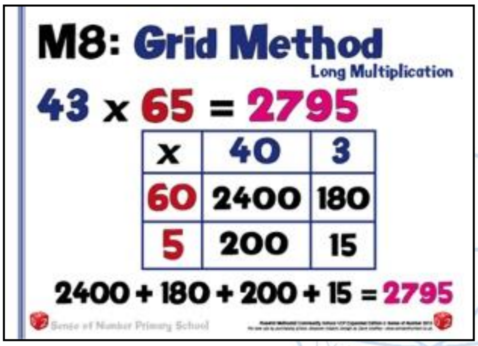
At this stage, using Place Value Counters rather than Base 10 enables the calculation to be created more efficiently.

Y5

Grid Multiplication

Extend the grid method to TU × TU, asking children to estimate first so that they have a general idea of the answer.

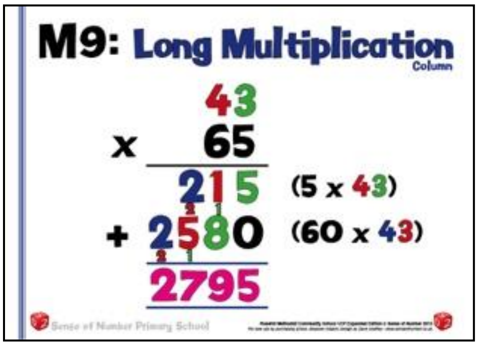
(43 × 65 is approximately 40 × 70 = 2800.)



As mentioned earlier, grid method is often the 'choice' of many children in Years 5 and 6, due to its ease in both procedure and understanding / place value and is the method that they will mainly use for simple long multiplication calculations.

Column multiplication

Children should only use 'standard' column method for long multiplication if they can regularly get it correct using this method.



There is no 'rule' regarding the position of the 'carry'digits.

Each choice has advantages and complications.

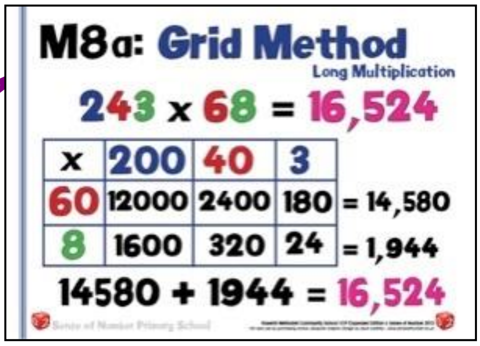
Either carry the digits mentally or have your own favoured position for these digits.

Y6

Again, estimate first:

243 × 68 is approximately 200 × 70 = 14000.

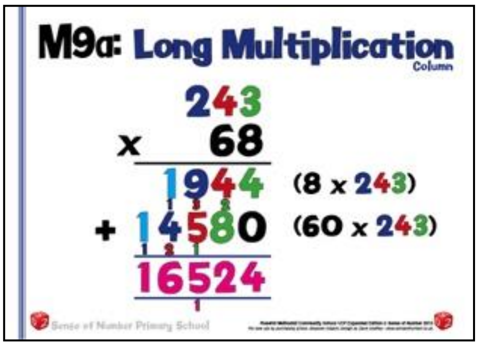
Grid Multiplication



For 3 digit x 2 digit calculations, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added.

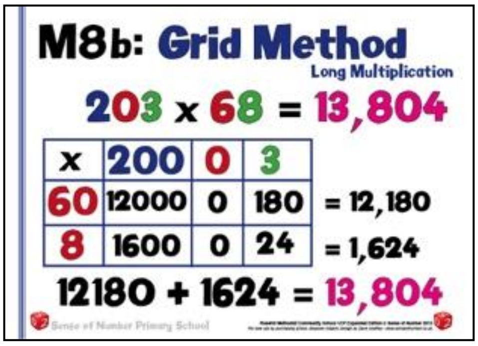
Use this method when you find the standard method to be unreliable or difficult to remember.

Column multiplication



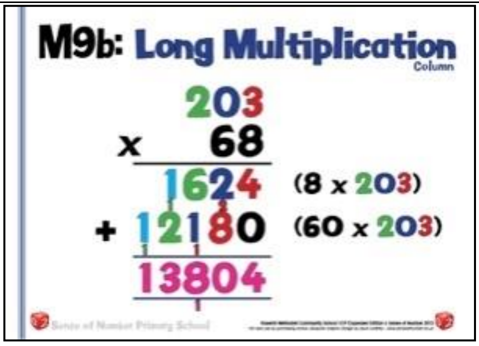
Most children, at this point, should be encouraged to choose the standard method. For 3 digit x 2 digit calculations it is especially efficient, and less prone to errors when mastered.

Although they may find the grid method easier to apply, it is much slower / less efficient.



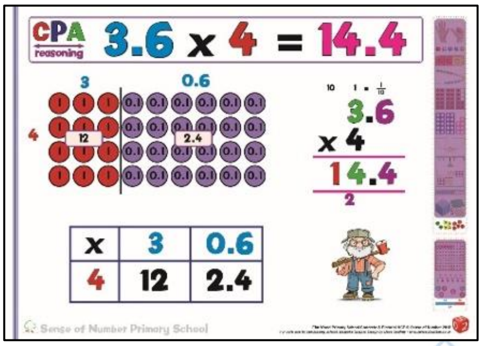
The Grid Method is an interesting way to show the part products that are created when multiplying with a 3 digit number with no Tens (or Zero in the Tens place).

The 4 parts are either very large (12000&1600) or quite small (180&24). As the method shows 6 parts altogether, it is clear that 2 of them are not used.

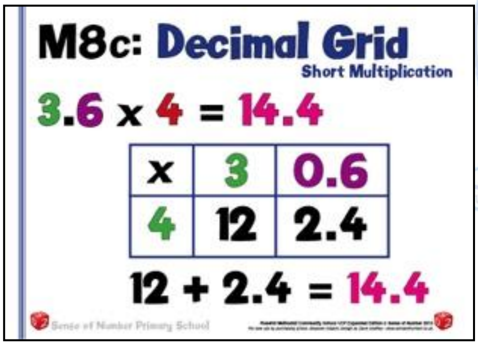


The column method shows the same overall product as the Grid Method (13,804)

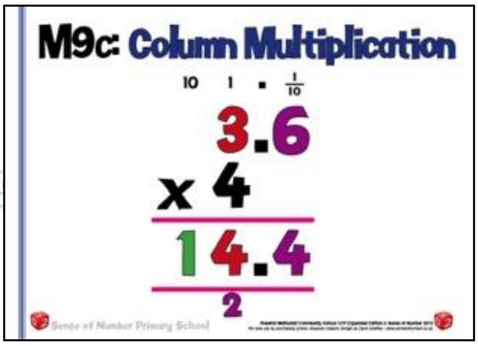
It doesn't, however, appear to be any different to a regular 3 digit x 2 digit calculation as the display simply shows the 2 part products created when multiplying by 68



Even when multiplying decimals, Place Value Counters can be used first in order to visualise the calculation before progressing onto Grid and Column Methods



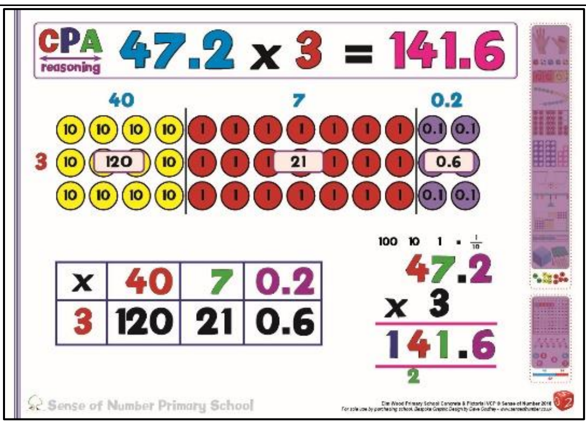
Many children will find the use of Grid method as an efficient method for multiplying decimals. They must also practice mental partitioning for decimal calculations such as the one above.

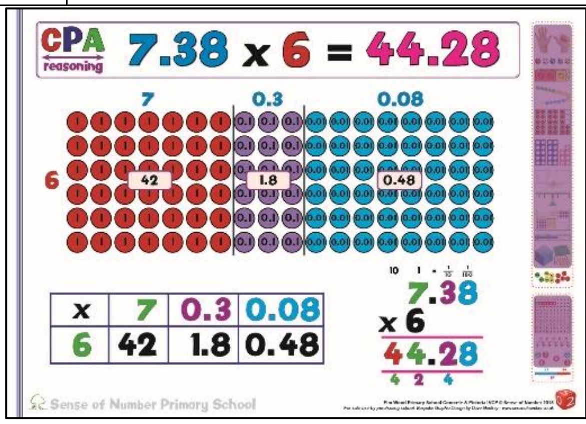


Extend column method into decimal multiplication.

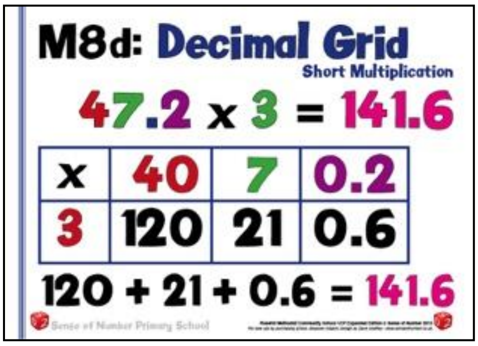
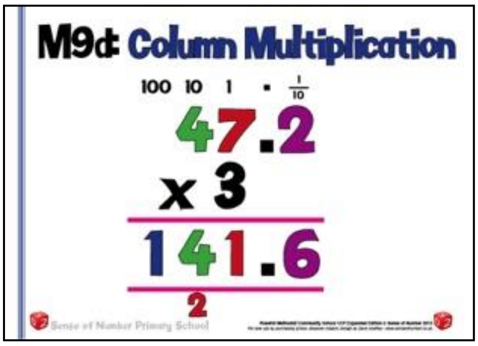
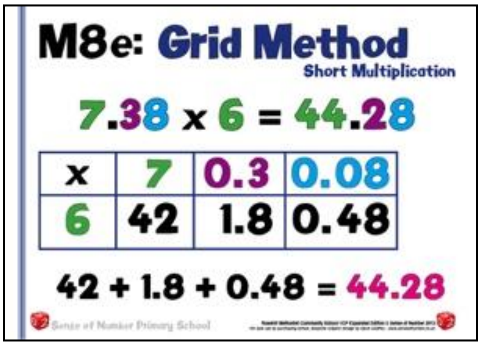
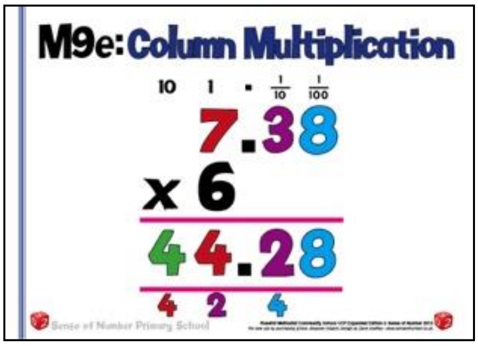
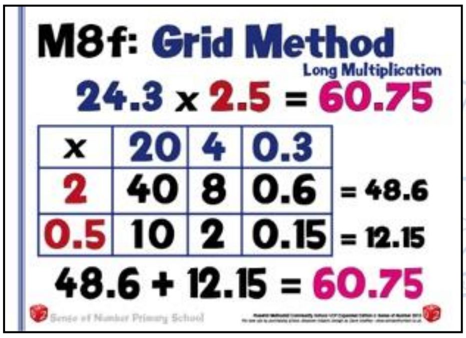
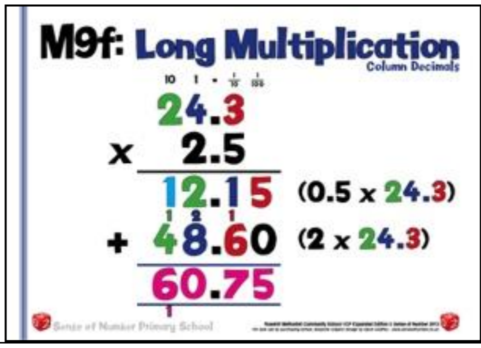
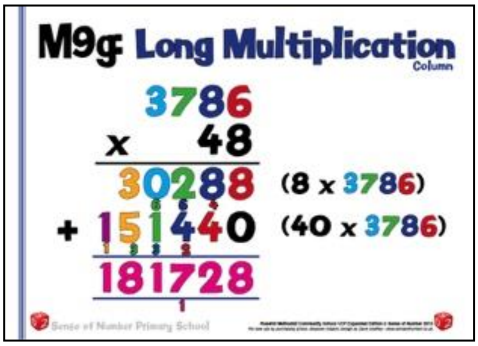
It is advisable to set out the columns as shown above so that the place value remains secure.

It is also helpful in showing how many digits will need to be displayed after the decimal point.

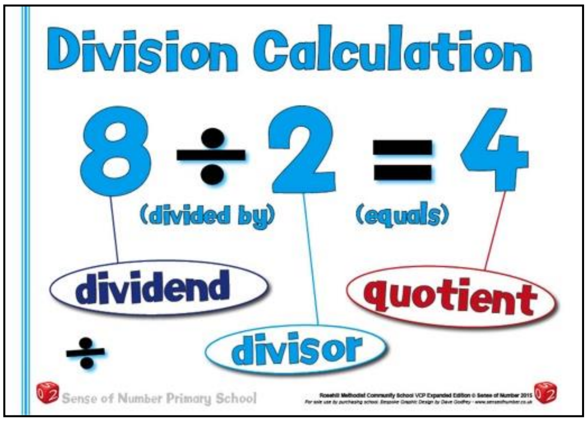




Even when the calculations become quite complex, Place Value Counters allow an instant visualisation, not only of the actual calculation but also of the exchanging / regrouping which needs to take place. This can really help to secure decimal place value within the calculation. Ask the children to make the calculation using Place Value Counters, and explain the procedure in stages before moving to column method.

Y6		
		 In the examples above, continue to think carefully about the layout of the calculation, keeping the place value accurate when multiplying.
		
	At this point children can use either standard method or grid method, but the column procedue tends to be more efficient.	
		 By the time children meet 4 digits by 2 digits, the only efficient method is the standard, column method for Long Multiplication.

Division Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in firstly developing clear understanding of division (including simple whole number remainders) through grouping and sharing, building up to written procedures. Unlike addition, subtraction and multiplication, there are no column methods, but we will explore both traditional written division (the 'bus stop' method) and 'chunking', firstly using short division 2 digits ÷ 1 digit, extending to 3 / 4 digits ÷ 1 digit, then finally exploring long division of 4 / 5 digits ÷ 2 digits.

To divide successfully in their heads, children need to be able to:

- **understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;**

For example, without a clear understanding that 72 can be partitioned into 60 and 12, 50 and 22, 40 and 32 or 30 and 42 (as well as 70 and 2), it would be difficult to divide 72 by 6, 5 4 or 3 using the 'chunking' method.

72 ÷ 6 'chunks' into 60 and 12

72 ÷ 5 'chunks' into 50 and 22

72 ÷ 4 'chunks' into 40 and 32

72 ÷ 3 'chunks' into 60 and 12 (or 30, 30 and 12)

- **recall multiplication and division facts to 12 × 12, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;**
- **know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;**
- **understand and use multiplication and division as inverse operations.**

Children need to acquire **one efficient written method of calculation for division**, which they know they can rely on **when mental methods are not appropriate.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successfully, children also need to be able to:

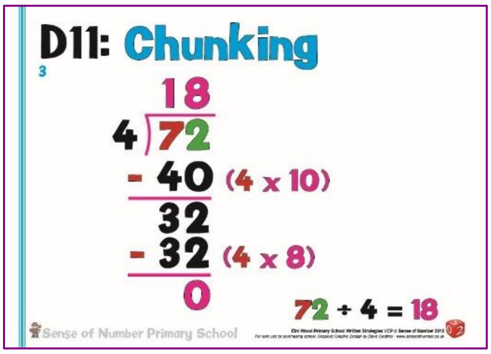
- **visualise how to calculate the quotient by visualising repeated addition;**
- **estimate how many times one number divides into another – for example, approximately how many sixes there are in 99, or how many 23s there are in 100;**
  (to do this children must again be able work comfortable with simple multiples of the divisor)
- **multiply a two-digit number by a single-digit number mentally;**
  (Check out the wide range of mental division processes from Page 68 onwards)
- **understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10.**
  E.g. 4 x 7 = 28 so 4 x 70 = 280 or 40 x 7 = 280 or 4 x 700 = 2800
  This will lead to 28 ÷ 7 = 4 so 280 ÷ *7 = 40 or 2800 ÷ 4 = 700*
- **subtract numbers using the column method (if using the NNS 'chunking' method)**

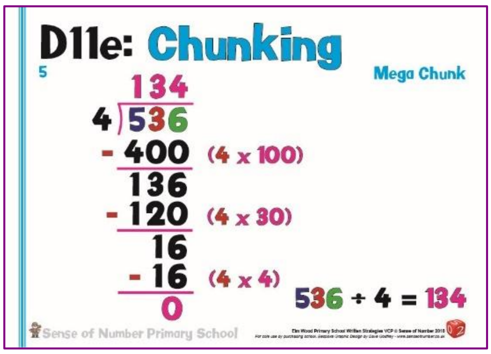
The above points are crucial. If children do not have a secure understanding of these prior-learning objectives then they are unlikely to divide with confidence or success, especially when attempting the 'chunking' method of division.

N.B. Please note that there are two different 'policies' for chunking, both of which are outlined and explained within this document.

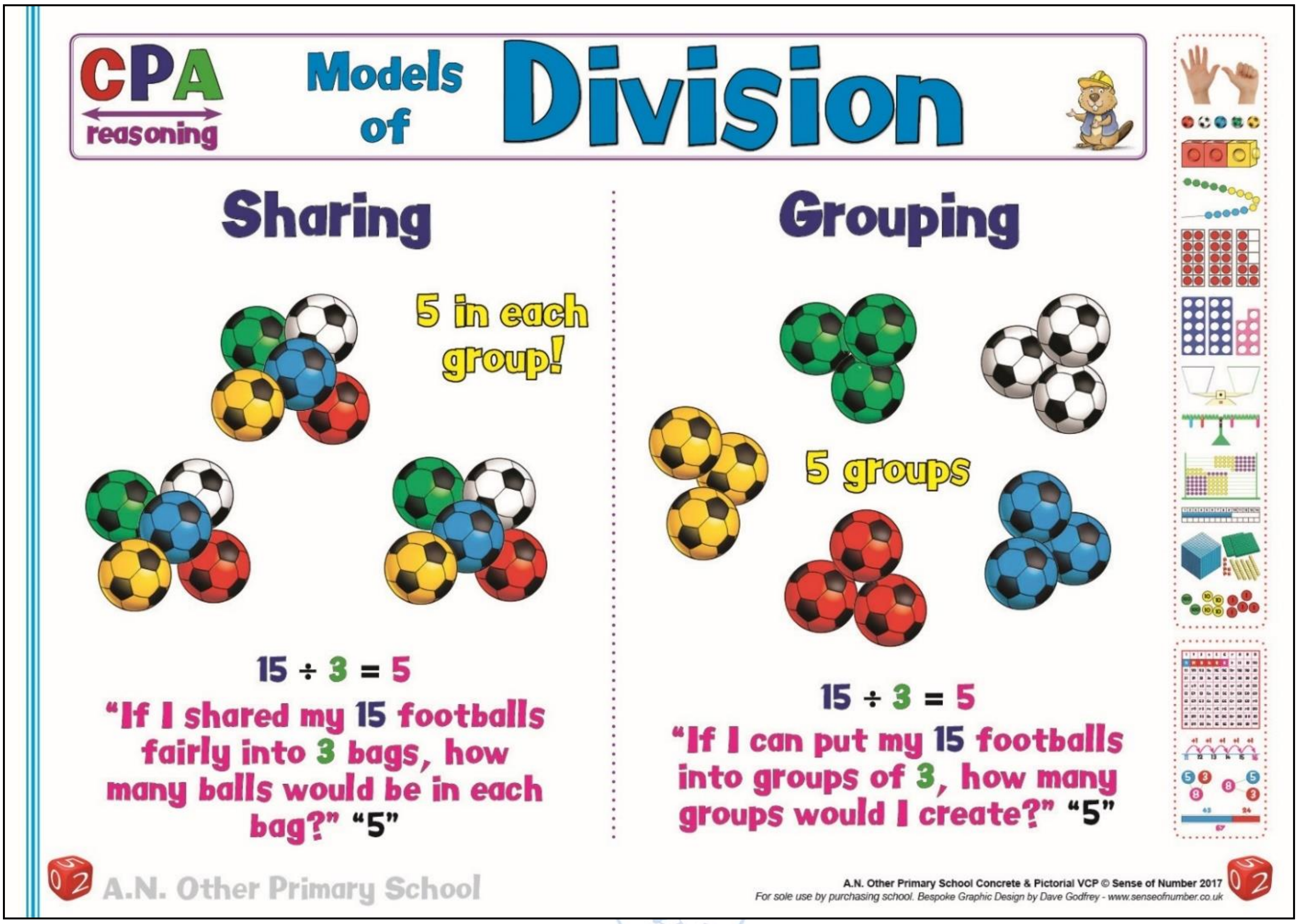
Chunking Option 1: National Numeracy Strategy Model:

The first would be used by schools who have continued to adopt the NNS model. This model was part of the curriculum for 15 years but was never embraced to the same level as the grid method of multiplication or the number line method of subtraction. Many schools found NNS chunking to be 'unwieldy' and very difficult for children to memorise, even though it was based on clear mathematical principles. There were often too many stages to 'chunking' and only children with a very secure knowledge of tables and a strong sense of number were able to use it consistently and accurately. It does, however, provide an alternative approach to long division which some children favour.





Models of Division



The poster above clearly shows that division can be viewed in two entirely different ways.

A calculation such as 15 ÷ 3 can mean one of two things: –
'**15 shared between 3** or '**How many 3s in 15?'**

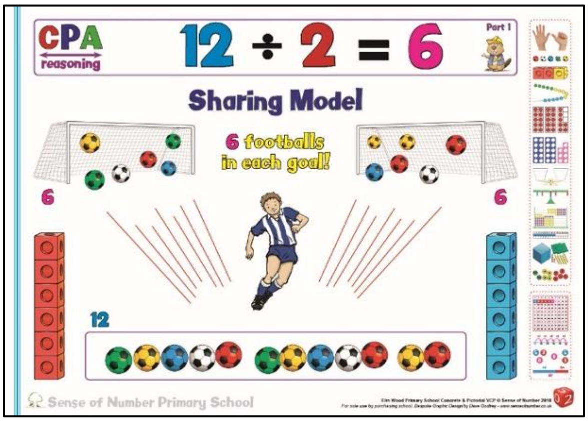
As the images show, if there are 15 footballs then they can be shared equally between 3 bags (with 5 in each bag) or groups of 3 footballs can be put into each bag (with there being 5 bags).

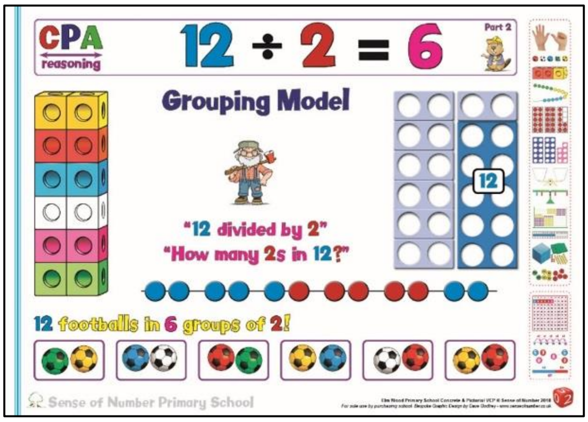
If the question is in a word problem format then the interpretation is specified, but if it is simply an abstract calculation then children can choose either meaning in order to answer the question.

It is **crucial** that children's early experiences of division allow them to model, demonstrate, explain, draw and practice answering division calculations using **<u>both</u>** interpretations.

They should be given calculations such as **20 ÷ 5, 16 ÷ 2, 12 ÷ 4 and 30 ÷ 10,** and asked to solve them practically then pictorially using both methods. Resources such as counters, cubes, pencils, balls, food, toys etc can be shared or grouped so that division is not seen as an abstract concept.

This means that they will always have two options open to them for any given division, allowing them to use tables facts and grouping for certain calculations, and equal sharing for others.





They will also realise that, in a similar way to subtraction (counting on or counting back), they aren't restricted to a single method or strategy.

There are **14 key strategies for jottings / written multiplication**, which support the children's understanding, and which can be developed across the year groups.

These can be seen on the left hand poster, and are explained in much more detail, with specific examples outlined, in the remainder of this section on **written multiplication**.

Division In Key Stage 1 – Should Grouping or Sharing Be The Default Model?

When children think conceptually about division, their default understanding should probably be Division is Grouping, as this is the most efficient way to divide and is the principle which is most commonly used in Key Stage 2.

The most common principle explored with children in KS1, however, is usually that of **sharing**. This 'traditional' approach to the introduction of division is to begin with '**sharing'** as it is seen to be more 'natural' for children to demonstrate, and is viewed as being easier to understand.

Many children see the division symbol and are encouraged to recognise it as the 'share' sign. Whilst this is one interpretation, it is absolutely crucial that it is **never** called the share sign.

The division symbol should always be given the name 'divide' and, as mentioned, be taught as both sharing and grouping

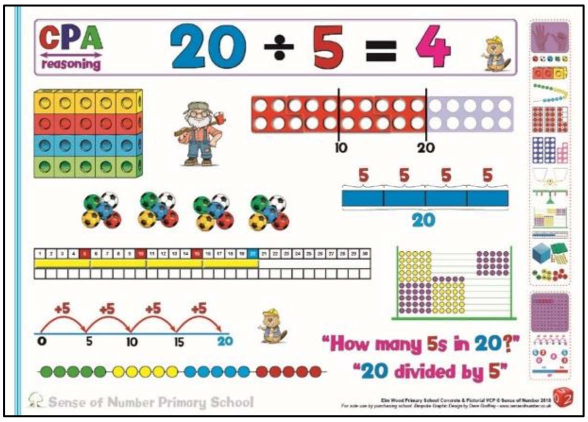
Children who are only given the 'sharing' interpretation of division often spend the majority of their time 'sharing' counters and other resources.

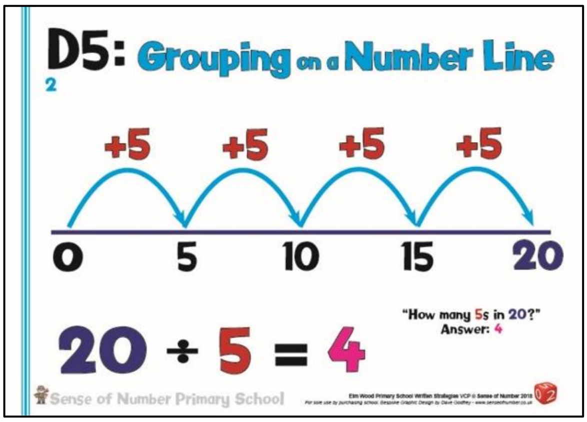
(i.e. **seeing 20 ÷ 5 as 20 shared between 5'**) – a rather laborious process which can only be achieved by counting, and which becomes increasingly inefficient as both the divisor and the number to be divided by (the dividend) increase)

These children are given little opportunity to use the grouping approach, which comes through exploring the inverse link to the repeated addition structure for multiplication

(E.g. If a child can work out **4 Fives**, either by **5 multiplied by 4 (5 x 4)** or **4 groups of 5 (4 x 5)** they can quickly solve **20 ÷ 5** if their first principle is '**How many 5's are there in 20?**')

This method is far more efficient and can quickly be achieved by counting in 5s to 20, something which most children in Y1 can do relatively easily.





Grouping in division can also be visualised extremely effectively using **Numicon, cubes, bead strings, abacuses and number rods**. **Number lines** can then be used as a quick **jotting** so that children can demonstrate the thinking process that they have used.

The only way to really visualise sharing is through counting things out one by one.

Grouping, not sharing, is the inverse of multiplication.

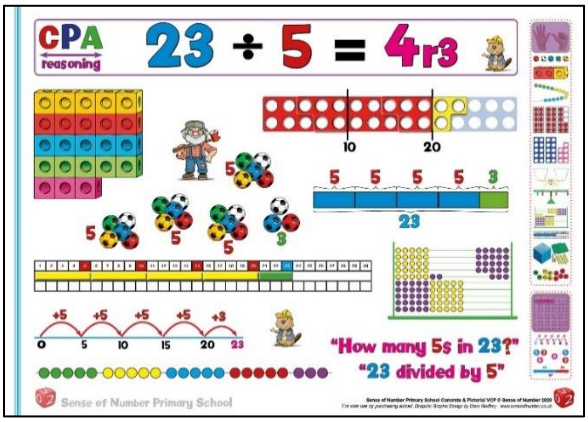
Sharing is division as fractions, and, as explored within the mental division section, does have its own set of strategies.

Once children have grouping as their first principle for division they can answer any simple calculation by counting in different steps (2s, 5s, 10s then 3s, 4s, 6s etc.). As soon as they learn their tables facts then they can answer immediately.

E.g. How much quicker can a child answer the calculations 24 ÷ 2, 35 ÷ 5, 70 ÷ 10 or even calculations such as 100 ÷ 20 or 300 ÷ 25 using grouping?

Children taught sharing as their first principle for division would find it very difficult to even attempt these calculations as they would need far too many resources and it would take a long time to count them all out in order to share them.

Children who have sharing as their first principle also tend to get confused in KS2 when the understanding moves towards 'how many times does one number 'go into' another'.



The use of whole number remainders is also far easier to explain when dealing with division by grouping.

As the examples on the poster display, the calculation **23 ÷ 5** can be clearly shown in a wide range of ways if it is viewed as **'How many 5s in 23?**' The Numicon, cubes and abacus give useful visualisations of the concept, the footballs allow it to be seen in a real life scenario, whilst the number line and number rods show it in a slightly more abstract, but still clearly explained way.

When children are taught grouping as their default method for simple division questions it means that they;

- secure understanding that the divisor is crucially important in the calculation
- can link to counting in equal steps on a number line
- have images to support understanding of what to do with remainders (Numicon)
- have a far more efficient method as the divisor increases
- have a much firmer basis on which to build KS2 division strategies

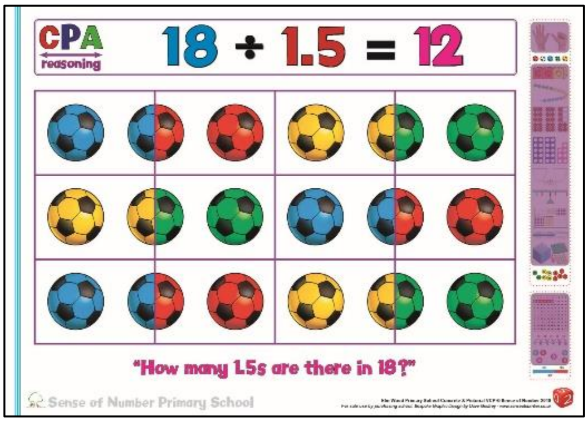
<u>Dividing With Fractions (Grouping & Sharing)</u>

As mentioned earlier, though, the most important teaching point is to make sure that **both** structures (**grouping <u>and</u> sharing**) are established immediately.

This means that even complex calculations involving fractions (as **either the dividend or divisor**) can be solved with confidence and accuracy.

Grouping – Fractions as the Divisor

As the image below shows, division as grouping not only allows regular calculations to be answered quickly by the use of tables facts (or by quickly counting up in different steps). It also enables children to visualise and answer higher level calculations such as **18 ÷ 1.5.**



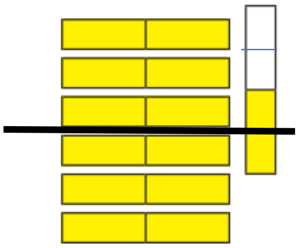
Using grouping we can ask the question '**How many 1.5s are there in 18?'**

The question can then be answered pictorially (or practically using number rods cubes or fraction circles) or simply worked out by counting in 1.5s to 18.

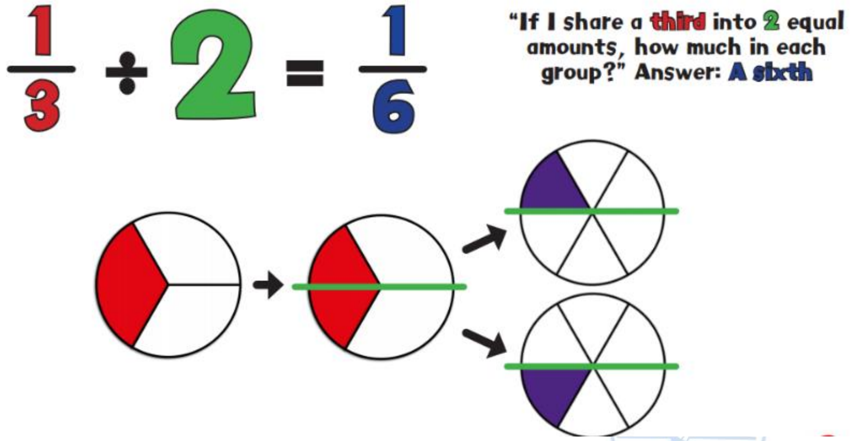
18 shared between 1 ½ or 1.5 would be almost impossible to imagine / visualise.

Sharing – Fractions as the Dividend
When the dividend is a fraction, and the divisor is a whole number, sharing is usually a far better structure to use when trying to solve the problem.

E.g. **6 ½ ÷ 2.**



If we imagine this as **6 ½** chocolate bars **shared between 2** people then each will get 3 ¼ bars.
Using grouping in this instance would lead to a far more complicated
(*How many 2s in 6 ½?)*



Even for a calculation such as **1/3 ÷2** (or ½ ÷3), we can take a third of a cake then share it between 2 people
(or a half of a cake and share it between 3 people)
so that each gets 1/6 of a cake.

3 equal pieces - or thirds - all shared into 2 equal pieces gives 6 equal pieces - or sixths
(2 equal pieces - or halves – both shared into 3 equal pieces gives 6 equal pieces – or sixths)

General Principles of Teaching Early Models of Division
Consequently, this policy is mainly structured around the teaching of mental and regular, simple division as grouping, moving from counting up in different steps to learning tables facts and eventually progressing towards the chunking methods of division in KS2.

Sharing is introduced as division in KS1, but is then taught mainly as part of the fractions curriculum, where the link between fractions and division is emphasised and maintained throughout KS2.

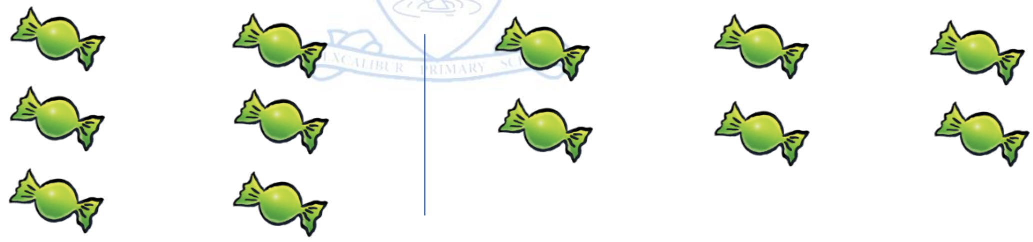
As mentioned, due to its efficiency, grouping is the default method for most simple division calculations.

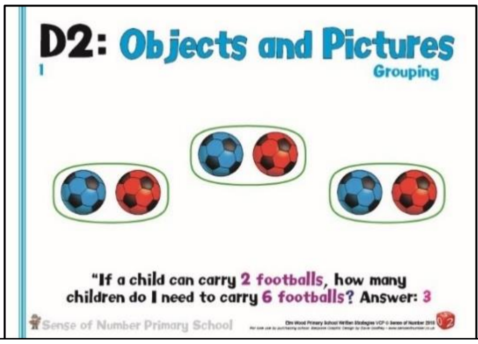
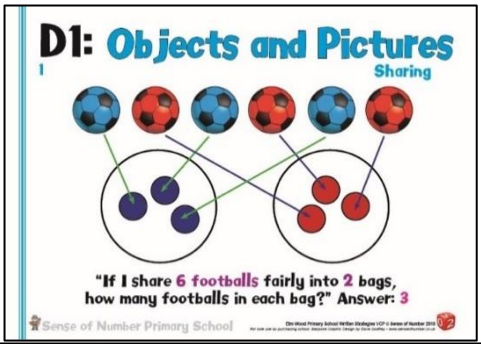
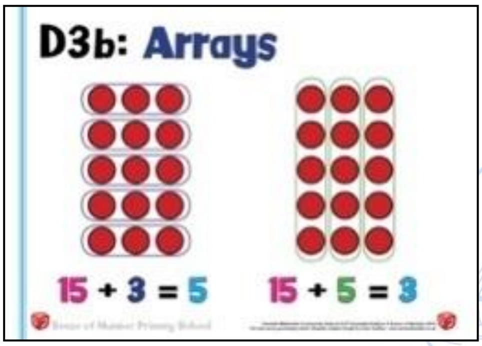
For conceptual understanding, however, when teaching formal 'bus stop' division, the sharing model is ideal for explaining how / why this method actually works. Therefore, this policy bases its written methods on both sharing (bus stop) and grouping (chunking)

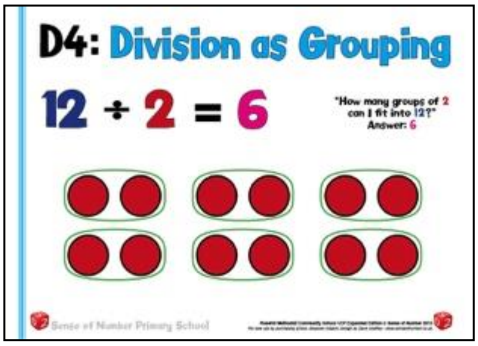
(See '**Written Methods of Division'** below for practical examples of bus stop division)

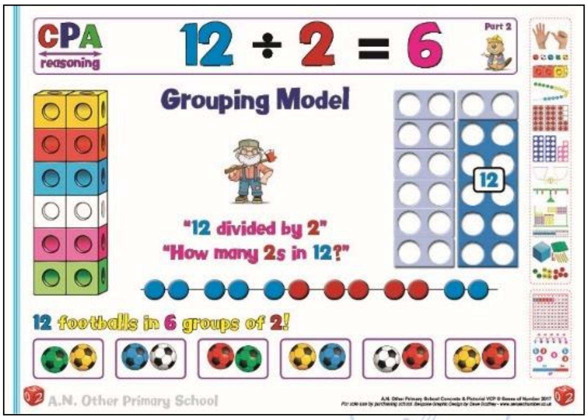
Written Methods Of Division

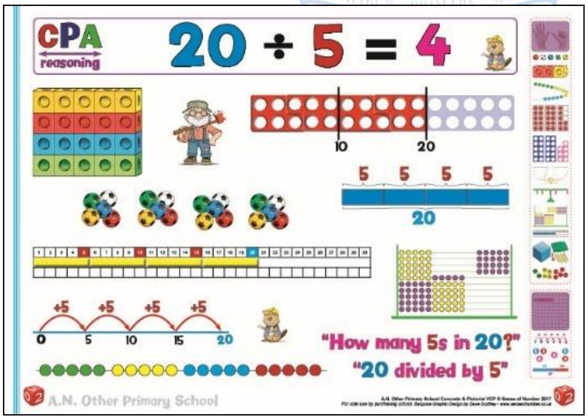
<table>
<tr><th rowspan="2">Stage 1</th><th colspan="2">Concepts and Number Lines (pre-chunking)</th></tr>
<tr><th>Grouping</th><th>Sharing</th></tr>
<tr><td>EYFS</td><td colspan="2">From EYFS onwards, children need to explore practically both grouping and sharing.
Links can then be made in both KS1 and KS2 between sharing and fractions.
For example, children can be asked to put their 12 dinosaurs into groups of 2, 3. 4 or 6.

The children could share 6 sweets between 2 or 3 people.
</td></tr>
<tr><td>Y1</td><td colspan="2">Begin by giving children opportunities to use concrete objects, pictorial representations and arrays with the support of the teacher.
Use the words ‘sharing’ and ‘grouping’ to identify the concepts involved.
Before using mathematical apparatus, use real objects and equipment such as cups, cakes, footballs, pencils, apples etc.
Children will firstly make then draw these objects by grouping and sharing them, working out the quotient by either counting the number of groups or the number within each group, then finally by writing the division statement.</td></tr>
</table>

	Make sure that the children have experience in representing the same division calculation in both ways for a range of different calculations. For example 6 ÷ 2 needs to be 'made' as both 6 in groups of 2 (How many 2s in 6?) and 6 shared between 2.  
Y2	Identify the link between multiplication and division using the array image. Begin to build on children's understanding that division is the inverse of multiplication, using arrays and number lines to support this thinking. Start to develop the use of the array to show that the same picture can be used to show both multiplication and division, and that the array demonstrates both sharing and grouping with one individual image.  As  with multiplication, make arrays using a wide range of objects, especially those which naturally occur in real-life such as windows, egg boxes, drawers or cake trays. Emphasise that all simple divisions can be worked out and pictured both ways (sharing or grouping) Encourage the children to see how the array makes it very easy to see both models of division, and practice circling the array depending on whether they want to show either: - • Grouping demonstrated both ways (see above) • Sharing demonstrated both ways (same pictures but opposite calculation written below each image) • The same calculation as either sharing or grouping (the second image in the first picture above as 15 ÷ 5 (grouping) or 15 ÷ 3 (sharing)

<table>
<tr>
<td></td>
<td>
Identify Grouping as the key model for simple division. Relate Grouping to knowledge of multiplication facts / the inverse of multiplication.
Regularly use the key vocabulary: 12 ÷ 2 means how many 2's can I fit into 12?'
or '20 ÷ 5 means how many 5's can I fit into 20?'




Begin to move away from real life objects / natural resources, instead using mathematical apparatus to show the same division calculation.


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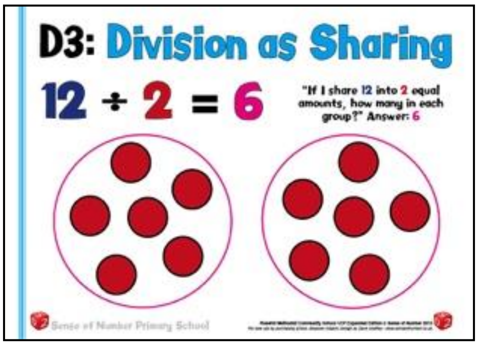
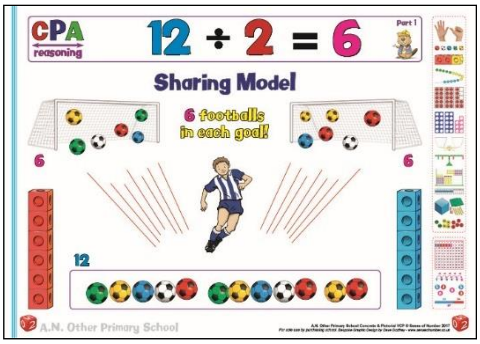
Identify Sharing as the secondary model of division.
Relate sharing to fractions, showing that sharing between 2 is the same as halving.




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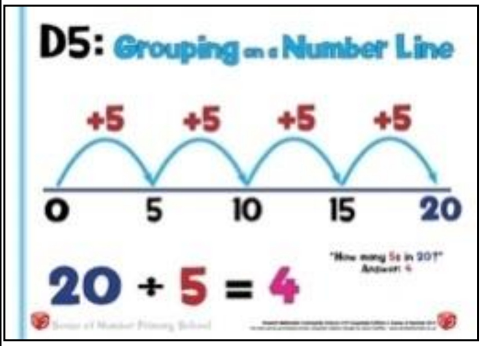
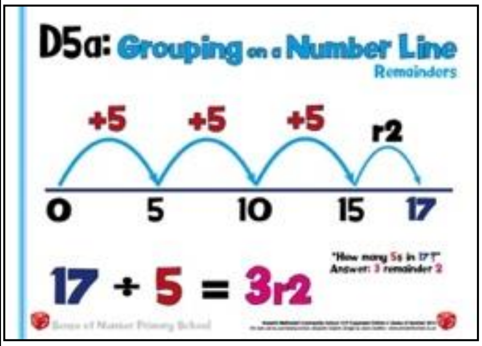




Moving towards a slightly more abstract visual, but one which still maintains clear understanding, the number line is an excellent way to quickly show how counting on (rather than counting back) is the easiest way to use a number line when solving a division calculation.

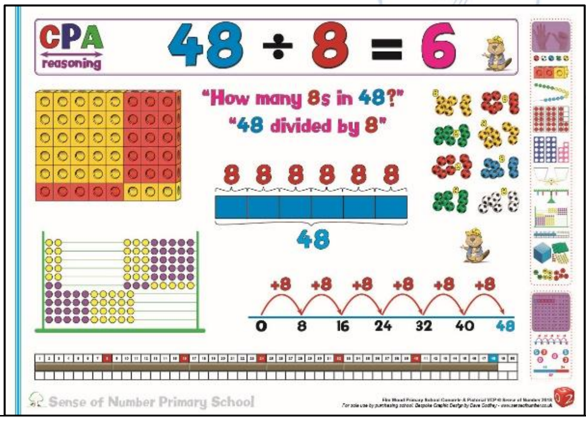
This is particularly beneficial if dealing with remainders. Using the example below, it is far easier to count forwards in 5s to 15 then count on 2 than starting at 17 and counting backwards in 5s.

Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.

Y3

It is really important to maintain the use of concrete materials as the calculations become gradually more complex, and tables facts harder to instantly recall.

Children using apparatus such as number rods, abacuses, Numicon and cubes can still explain the division and have a better understanding of the calculation if they are asked to make it.



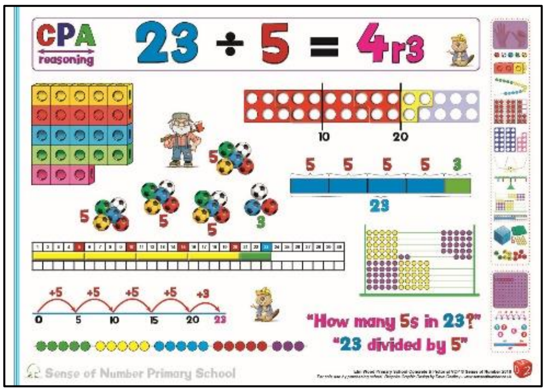
Continue to give children practical images for division by grouping when dealing with simple remainders.

Asking the question '***How many 5s are there in 23?'*** allows the opportunity to visualize the remainder.

The images of the cubes or the abacus below show 4 complete groups of 5 and a remainder of 3.

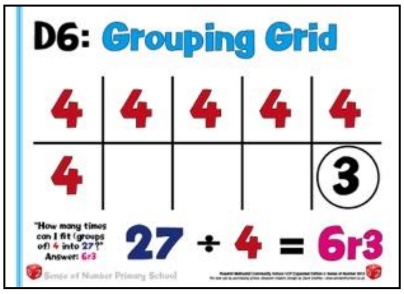
The Numicon image allows the '5' and '3' pieces to be placed over three '10' pieces and see how many 5s make 20, along with the additional 3.

The number rod image is almost a practical representation of a number line, especially when placed on the number track.



Division as grouping both with and without remainders can also be shown very effectively using children.

For 27 ÷ 4: - In the hall, use PE mats and ask children to move into groups of 4. The remainder go into a hoop.

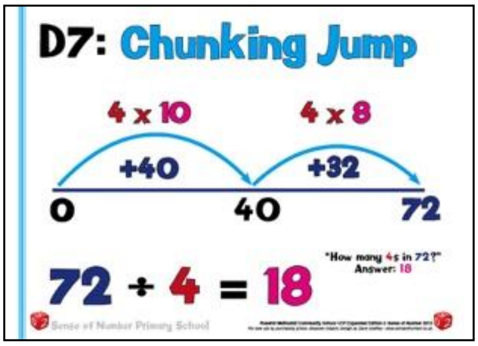


The image on the left (Grouping Grid) can then be used back in class, with the 4s being the mats and 3 the remainder in the hoop.

Stage 2	Chunking & Standard Methods	
	Standard Methods	**Chunking** **Find the Hunk &** **NNS Chunking**

Y3

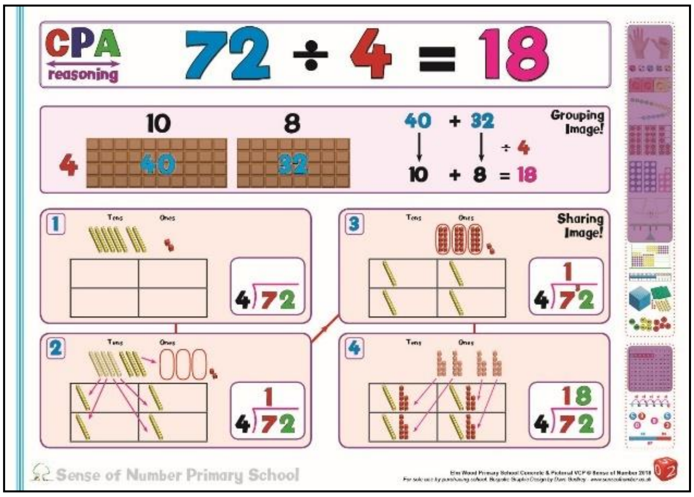
As



previously encountered in Y2, developing an understanding of division with the number line is an excellent way of linking division to multiplication. It can show division both as repeated subtraction, but it is simpler to show division by counting forward to find how many times one number 'goes into' another.

These slides introduce the Short Division (Bus Stop) method in Year 3.

It is recommended that when children start to use this strategy, it is only introduced once tables facts are relatively secure.

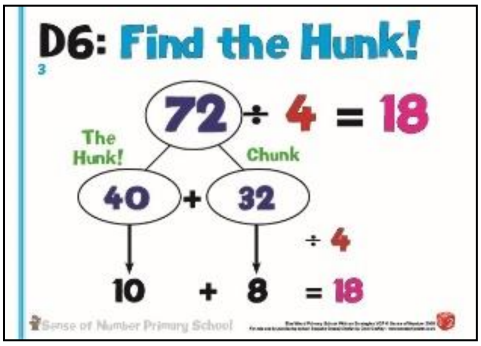


When introducing Short Division formally, use Base 10 and make sure you introduce it using the **sharing model (**as mentioned in the 'Teaching Short Division' explanation on the previous two pages)

The calculation starts with, 'I have 7 Tens, to share between 4. That's 1 Ten each with 3 remaining. These 3 Tens are regrouped into 30 Ones. The 32 Ones are now shared between 4– that's 8 Ones each.

The alternative approach to written division, which should be taught alongside the 'bus stop' method so that it can be used for examples which can be easily accessed, is known as 'Find the Hunk'.

Find the Hunk', as explained earlier, is really a



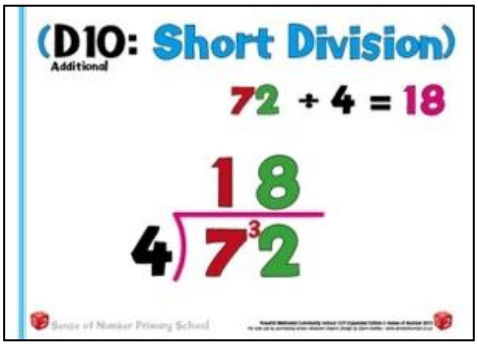
mental strategy based on partitioning in different ways. For a detailed overview of 'Find the Hunk' please read the section on mental division.

It can also be used as a quick written jotting to suppport mathematical thinking, enabling children to deal with the calculation quickly

For the example above, the Hunk is defined as being 10 times the divisor.

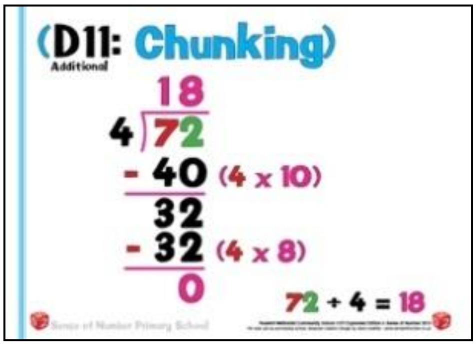
The divisor is 4, so the Hunk will be 4 x 10 = 40. This leaves a chunk of 32 remaining.

Both chunks are then divided by the divisor and then the answers to each totalled.



As mentioned previously, children should be taken to the standardised ‘bus stop’ format for short division only when they have mastered the use of apparatus and are able to explain the process.

At any stage they can revert back to the use of concrete materials to recap / aid their understanding, or to deal with higher level calculations in later year groups



‘Find the Hunk’ is a mental strategy based on simply partitioning numbers in different ways.

The National Numeracy Strategy chunking method, however, is based on subtraction / repeated addition.

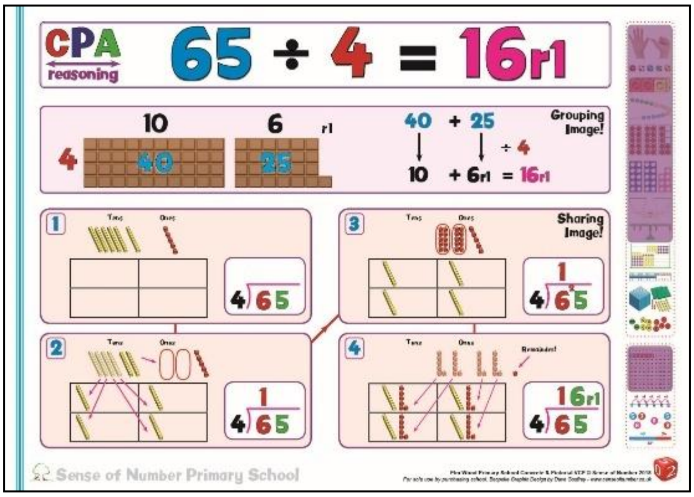
In the example above, 40 (4 x 10) is initially subtracted from the dividend, followed by 32 (4 x 8). This means that altogether 4 x 18 has been subtracted, making 18 the answer. This method requires the children to be secure in their division facts, to be able to carry out column subtraction effectively, to write the separate quotients as multiplication statements and also to be able to remember a quite complex layout.

This strategy can be unwieldy, somewhat confusing and relatively difficult to master. Therefore, the recommendation within this to use Find the Hunk as the default strategy for chunking and only to adopt NNS chunking when tackling complex long divisions.

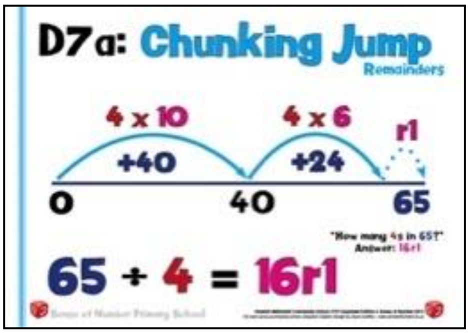
Once short division of 2 digit numbers with a whole number quotient has been mastered, begin to practise examples (using both ‘bus stop’ and chunking) of short division where the quotient includes remainders.

The examples below are explained in exactly the same way as the previous example but the remainder is simply left as a whole number.

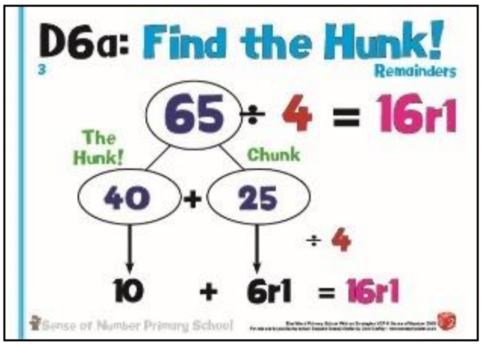
(In later years it will be expressed as a fraction or a decimal for a more accurate answer)

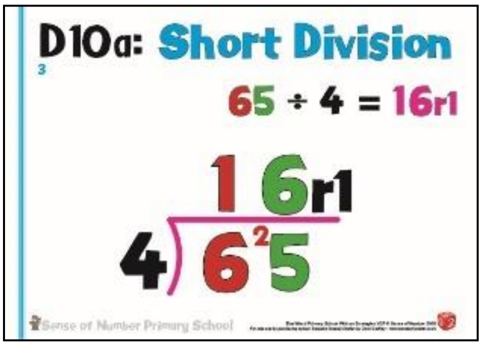


Base 10 will still be used for the first principles of Concrete – Pictorial – Abstract before the children progress to the abstract method.

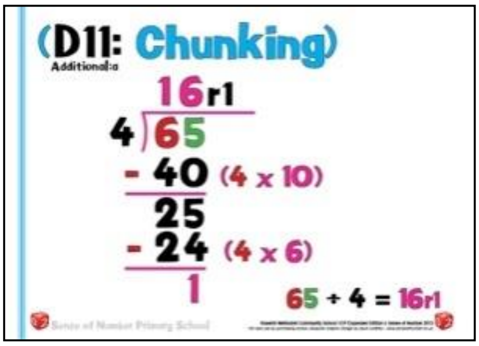


The number line provides an excellent image to show division with remainders, and can be used to support / instead of ‘Find The Hunk’. Find The Hunk is equally simple to use with remainders (see below)



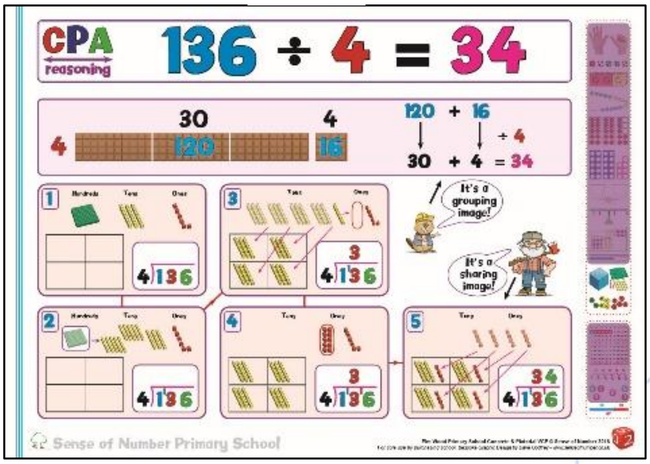


Clear links can now be made to the concrete images explored earlier, so that children understand the method



NNS Chunking remains a fairly tricky method to secure, especially remembering the steps

Y4



As described in more detail earlier, short division can now be developed into 3 digit dividends. Base 10 (or place value counters for children who have better understanding) is used tomodel and explain how the method works.

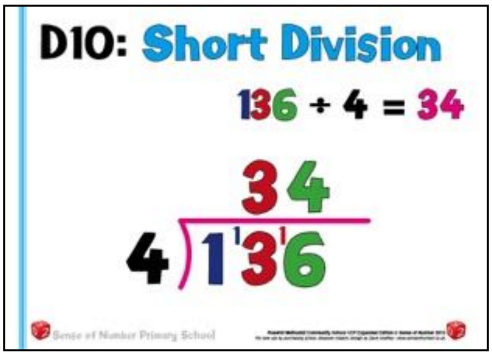
The calculation starts with, 'I have 1 Hundred, which cannot be shared (in Hundreds) between 4.

The Hundred is regrouped into 10 Tens.

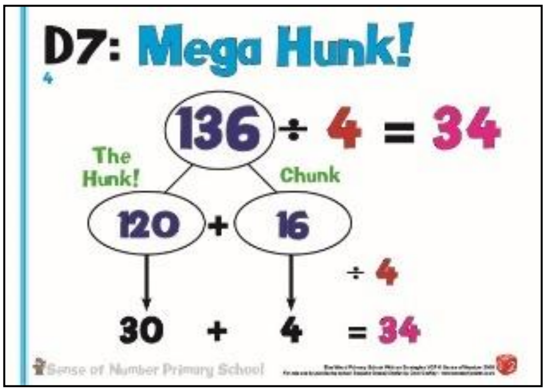
The 13 Tens are now shared between 4, giving 3 each.

The remaining Ten is regrouped into 10 Ones.

The 16 Ones are now shared between 4 –4 Ones each.



As before, once understood and explained with a range of calculations, children use the 'bus stop' method.



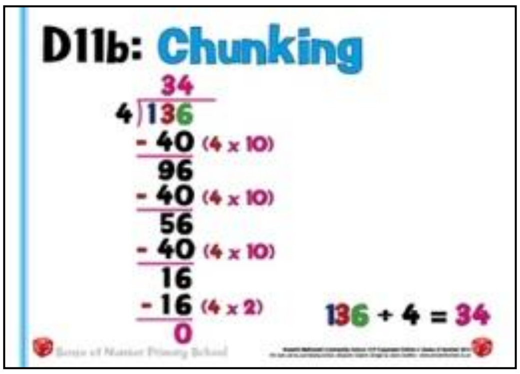
Mega Hunk' is the natural development of the '**Find the Hunk'** strategy, simply using bigger chunks when the dividend is much larger.

In the example above, Mega Hunk is defined as being multiples of the Hunk (which is usually 10 times the divisor) .

The divisor is 4, so the initial Hunk will be 40. The 'Mega Hunk' will be 40 x 3 = 120, leaving a remaining chunk of 36.

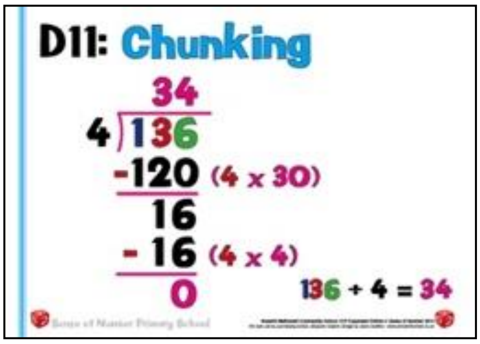
Again, both chunks are then divided by the divisor and then the answers totalled.

The



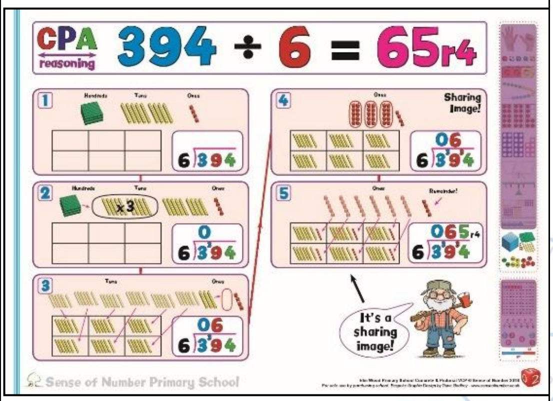
National Numeracy Strategy chunking method is also based on multiples of 10 times the divisor. The example above is an expanded version of the example below.

At first, children are asked to keep subtracting 10 times the divisor (40) until the remaining number is less than 40.



This can be extremely laborious and is open to multiple errors in calculation or layout, so children are then encouraged to look for bigger chunks that can be subtracted. The example above uses exactly the same chunks as ‘Mega Hunk’ but is a more complex layout.

Y5

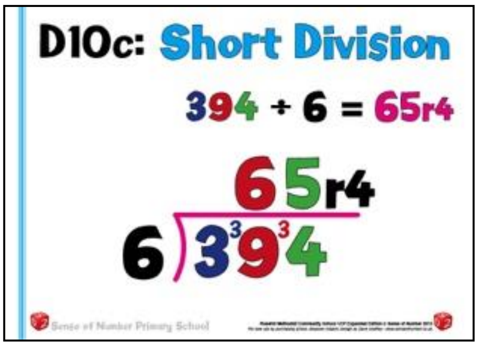


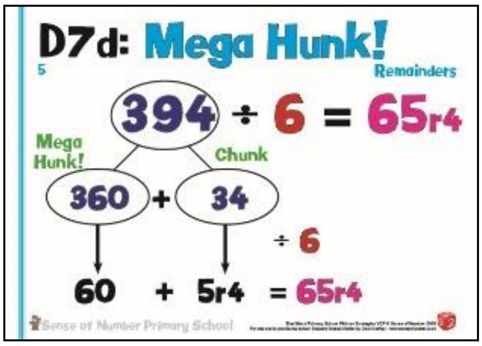
Continue to use apparatus to show division with remainders, and also where larger amounts of regrouping is needed.

In the example the 3 Hundreds can’t be shared between 6 so they are regrouped into 30 Tens.

The 39 Tens are shared between 6, giving 6 each.

The remaining 3 Tens are regrouped into 30 Ones. The 34 Ones are shared between 6, giving 5 each with a remainder of 4

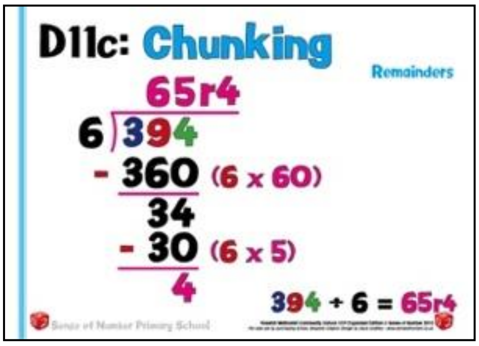




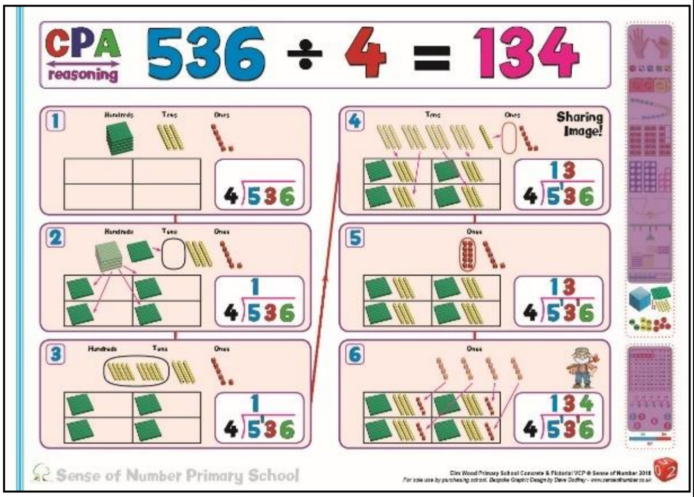
Continue to use the Find the Hunk strategy whenever possible.

Even with remainders it becomes a very efficient mental jotting. With the example above 394 can be regrouped as 360 + 34.

Both chunks can then quickly be divided by 6, leaving a remainder of 4.



By this stage children using the NNS method should be finding much larger chunks of the divisor, making the method much easier to complete (although still less efficient than Find the Hunk)



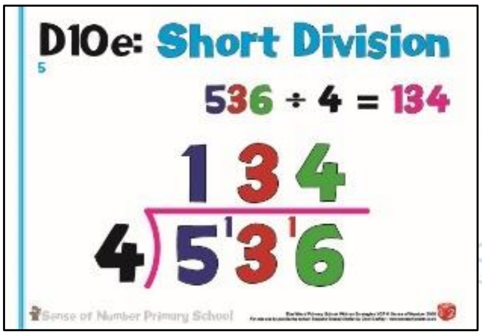
When the Hundreds can be shared between the divisor, it is beneficial to still use concrete materials, allowing children to actually regroup / exchange the Hundreds and Tens.

In the example above, the 5 Hundreds can be shared (in Hundreds) between 4, giving 1 each.

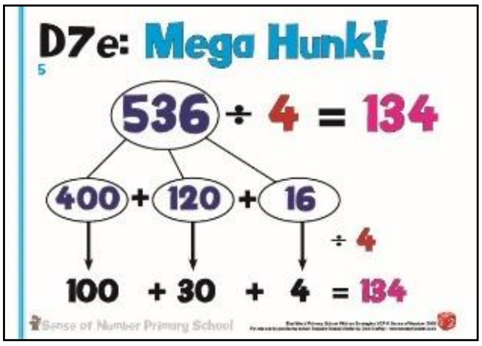
The remaining Hundred is regrouped into Tens.

The 13 Tens are shared between 4, giving 3 each. The remaining Ten is regrouped into Ones.

The 16 Ones are shared between 4, giving 4 each.



Once a child is confident with the apparatus then, as before, they can use the 'bus stop' method, explaining it when necessary.

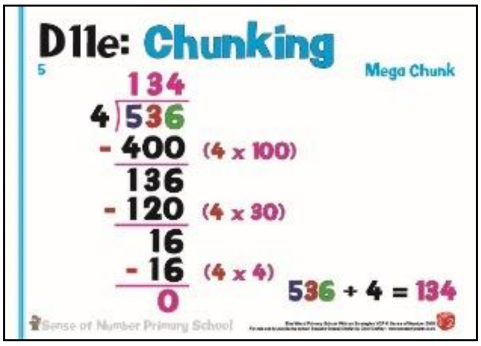


For this type of division, there are 3 'hunks'.

536 is regrouped / partitioned a different way into a multiple of 400 (400), a multiple of 40 (120) and a multiple of 4 (16).

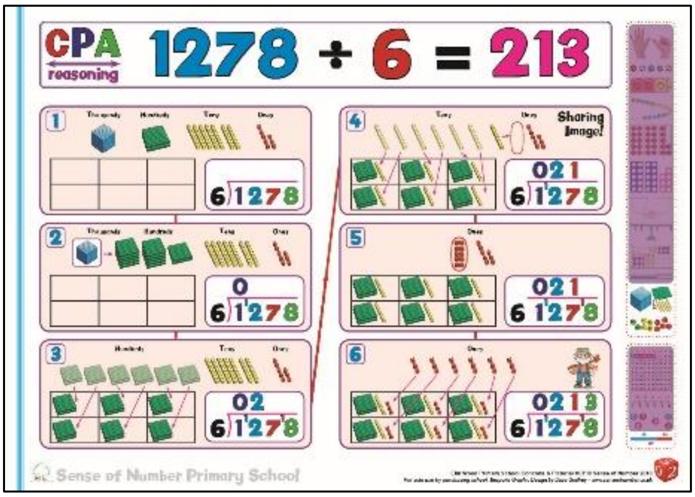
Each of these 3 numbers is then divided by 4.

Children who have practiced 'Mega Hunk' for smaller numbers and are confident in their tables facts still find this a fairly simple method to complete.



The NNS Chunking method, in effect, partitions the 536 in the same way as Find The Hunk.

The need for subtraction and the complex layout makes it more difficult to remember without a very clear understanding, but it does allow children to practice a method that they may prefer once they are asked to do long division.



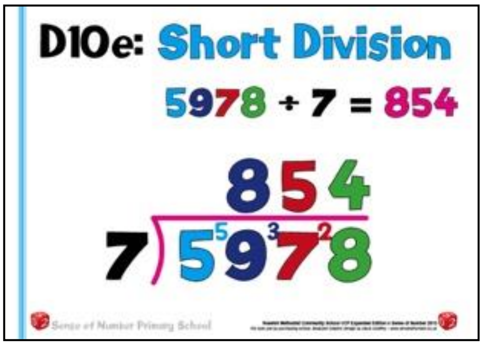
The final stage in which apparatus would usually be used is for simple 4 digit short division calculations.

Once children can explain a 4 digit calculation using concrete materials then they are ready to use 'bus stop' method for any calculations of 3 digits and above.

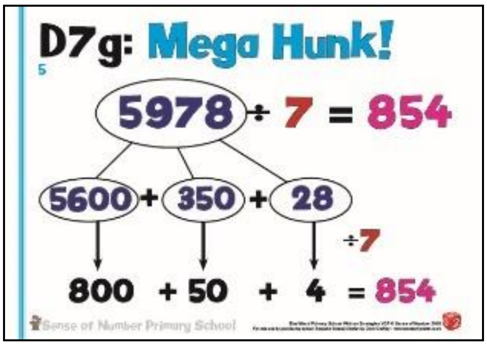
If the example on the left was to be completed using 'Mega Hunk' then it would be a very simple: -

1278 ÷6 = 213

1200	60	18	÷6
200	10	3	

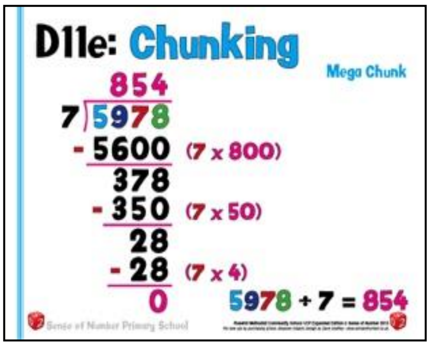


At this point the numbers would require far too much apparatus for the calculation to be viable visually so children would simpy use a 'bus stop' method. If at any point,however, they were asked to explain / reason their answer, they would be able to use the understanding developed through the use of Base 10.



Even with 4 digit numbers, Mega Hunk is relatively simple for children with a good sense of number.

5978 can be regrouped into 5600, 350 and 28. These numbers can then be divided by 4.

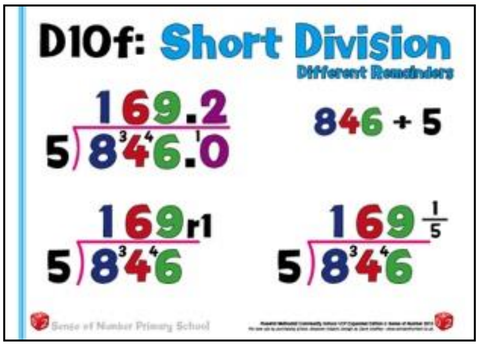


NNS chunking, as before, displays the same numbers but in a slightly more difficult layout.

Children should develop the ability to represent the quotient initially as a straightforward remainder (given as a whole number) , but also as a decimal or fractional remainder.

The examples of short division below show the whole number remainder in the Mega Hunk & NNS examples, but also the fractional and decimal remainders in the 'bus stop' examples.

Y6

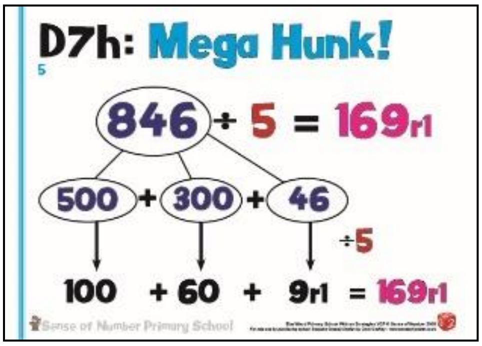


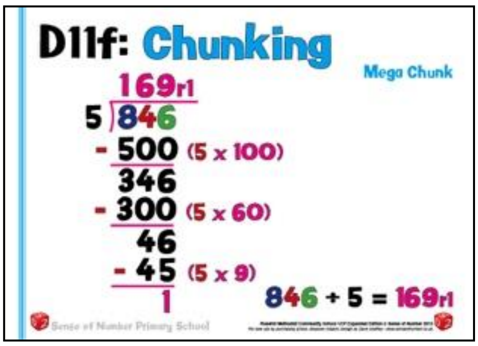
Once a child is in Upper Key Stage 2 they should be aware that giving a remainder as a whole number is not accurate enough.

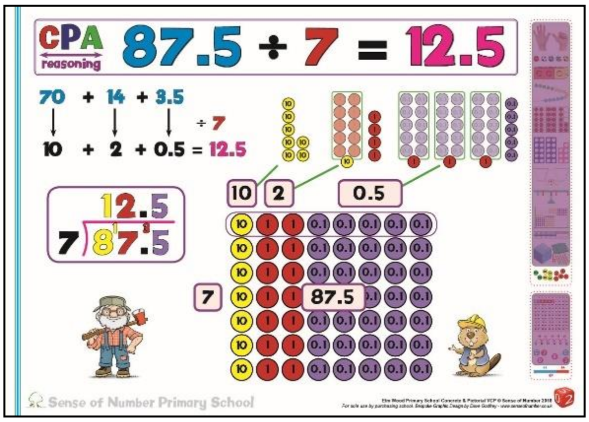
A remainder of 1, for example, has a completely different meaning when the divisor changes.

When dividing by 2 it represents ½ (or 0.5), when dividing by 4 it is ¼ (or 0.25), when dividing by 10 it is worth 1/10 (or 0.1).

In the example above, when dividing by 5, the remainder represents / is worth 1/5 (or 0.2).

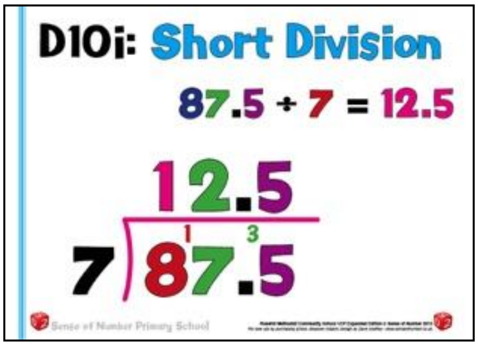


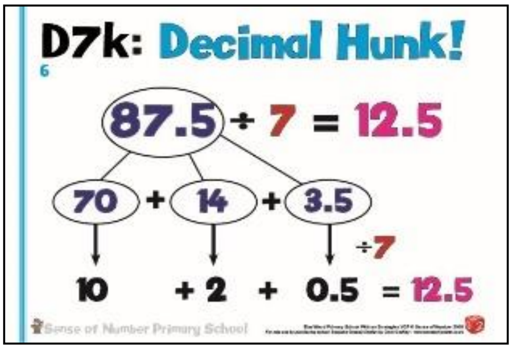




If necessary, place value counters can be used for children to explain short division using decimals.

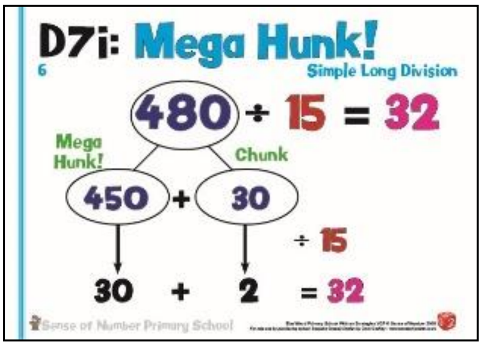
In the example above, the 8 Tens, 7 Ones and 5 Tenths have been regrouped into 7 Tens, 14 Ones and 35 Tenths, each of which can be divided by 7.





Decimal Hunk allows the children to represent a relatively difficult calculation in an efficient way, one that can be answered very quickly.

If 87.5 is regrouped into 70, 14 and 3.5, each number can then be divided by 7, and the quotient determined immediately.

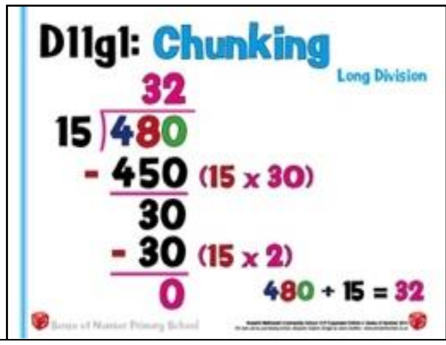


When introducing simple long division questions, where the multiples of the divisor are fairly easy to work out, it is often easier to find a quotient using the Mega Hunk strategy.

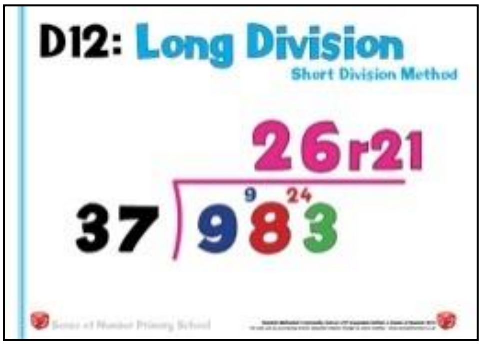
The eg is no more difficult than many short divisions.

As with some of the earlier examples, NNS chunking can be unwieldy if the smaller chunk is repeatedly subtracted (see above).

The method is much more efficient with a larger chunk subtracted (see below).



There are three different ways of calculating a more complex long division – continuing to use the short division method but with bigger numbers, learning the traditional method or using NNS Chunking. Find the Hunk is no longer an efficient strategy as the numbers are too big to work with efficiently.

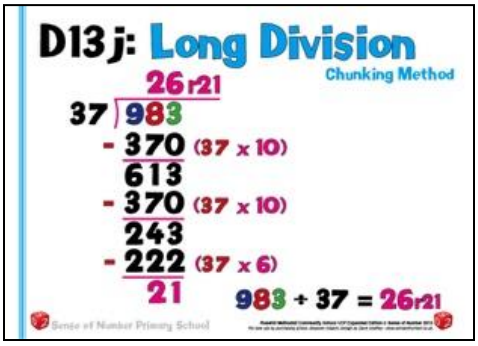


Only children who are extremely proficient with mental calculation would use the short division approach. It requires them to be able to work out a mental difference accurately, and often to 'carry' over a fairly substantial amount.

In the example above 98 Tens would be shared between 37, giving 2 each.

The remaining 24 Tens would be regrouped into 240 Ones.

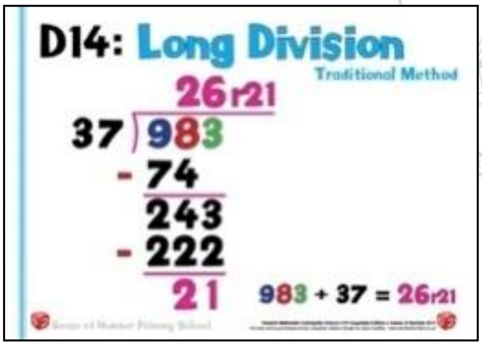
The 243 Ones are then shared between 37, giving 6 each with a remainder of 21.



It is only at this final stage where NNS Chunking becomes the preferred method.

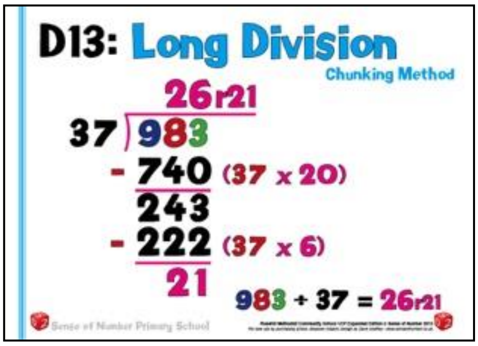
Unlike the traditional method (see the left hand column), where it is difficult to explain the method using place value, the NNS method maintains place value.

In the extended example above, the chunk of 370 is repeatedly subtracted, followed by a chunk of 222.



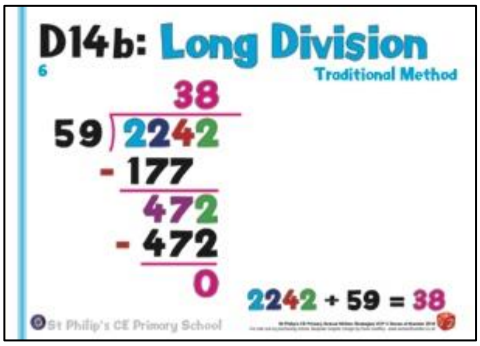
Most schools now adopt the traditional long division method, where multiples of the divisor are subtracted, and digits are 'dropped down' to form a new calculation.

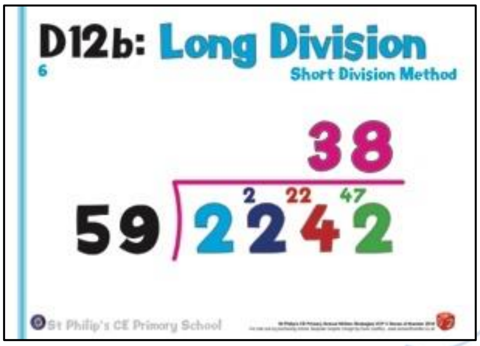
This can be a complicated method to remember for many pupils as it is very difficult to explain and doesn't really require place value. It relies on them being able to memorise the method.



The 'abridged' or shortened NNS chunking method is probably the most sensible and efficient way to approach long division.Using larger complete chunks of the divisor means that the children can see exactly what is being subtracted and don't have to rely on 'dropping down' any of the digits. Looking at this example alongside the one on the left, it is clear that NNS Chunking is almost the same methods as traditional long division, but with secure place value (i.e. 740 being subtracted rather than 74).

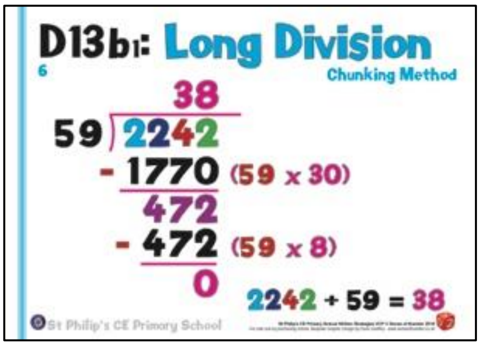
The examples below feature a 4-digit dividend.





The examples below feature a 4-digit dividend.

This would be the recommended Sense of Number method for long division.



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